Recitation 8

Chapter 22

Problem 34. *Two long, parallel conductors, separated by $r = 10.0$ cm, carry current in the same direction. The first wire carries current $I_1 = 5.00$ A, and the second carries $I_2 = 8.00$ A. (a) What is the magnitude of the magnetic field B_1 created by I_1 at the location of I_2? (b) What is the force per unit length exerted by I_1 on I_2? (c) What is the magnitude of the magnetic field B_2 created by I_2 at the location of I_1? (d) What is the force per unit length exerted by I_2 on I_1?*

(a) From Ampere's law, the B field generated by a long, thin current is

$$B = \frac{\mu_0 I}{2\pi r} \tag{1}$$

Plugging in I_1, we have

$$B_1 = \frac{\mu_0 I_1}{2\pi r} = 10.0\ \mu\text{T} \tag{2}$$

This B field depends on your distance from I_1, but because the wires are parallel, the B field from I_1 is constant along I_2 We can use the right hand rule to determine that $\mathbf{B}_1$ is perpendicular to both I_1 and r.

(b) From $F_B = q\mathbf{v} \times \mathbf{B}$ we have the force on a current carrying wire in a uniform magnetic field as

$$F_B = I\mathbf{l} \times \mathbf{B} \tag{3}$$

Combining these two equations, we have the force per unit length of I_1 on I_2 as

$$F_{B12}/l = I_2 B_1 = \frac{\mu_0 I_1 I_2}{2\pi r} = 80.0\ \mu\text{N} \tag{4}$$

where there is no $\sin\theta$ term in the cross product, because B_1 is perpendicular to I_2. By drawing the situation and doing some right hand rules, you can convince yourself that this force is *attractive*.

(c) Because the situation in (c) is identical to (a) with $I_1 \leftrightarrow I_2$, we simply relabel eqn. 2.

$$B_2 = \frac{\mu_0 I_2}{2\pi r} = 16.0\ \mu\text{T} \tag{5}$$

(d) Eqn. 4 is identical under the relabeling, so we have another attractive force at the same magnitude

$$F_{B21}/l = 80\ \mu\text{N} \tag{6}$$

as we would expect from Newton's third law (for every action there is an equal and opposite reaction).

Problem 37. *Four long, parallel conductors carry equal currents of $I = 5.00$ A. Figure P22.37 is an end view of the conductors. The current direction is into the page at points A and B and out of the page at points C and D. Calculate the magnitude and direction of the magnetic field at point P, located at the center of the square of edge length $a = 0.200$ m.*

First, let us pick a coordinate system by choosing unit vectors. Let $\hat{\mathbf{i}}$ be down and to the left, $\hat{\mathbf{j}}$ be down and to the right, and $\hat{\mathbf{k}}$ be straight down.

Using the right-hand rule, we determine the direction of the magnetic field at P generated by each wire to be

$$\widehat{B_A} = \hat{\mathbf{i}} \tag{7}$$
$$\widehat{B_B} = \hat{\mathbf{j}} \tag{8}$$
$$\widehat{B_C} = \hat{\mathbf{i}} \tag{9}$$
$$\widehat{B_D} = \hat{\mathbf{j}} \tag{10}$$

The magnitude of each B is given by

$$B = \frac{\mu_0 I}{2\pi r} \tag{11}$$

And since the currents have the same magnitude, and each corner is equidistant from the square center, each magnetic field contribution will have the same magnitude. The distance r is given by

$$r = \sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2} = \frac{a}{\sqrt{2}} \tag{12}$$

We still have to add our vector fields, which gives

$$\mathbf{B}_P = \mathbf{B}_A + \mathbf{B}_B + \mathbf{B}_C + \mathbf{B}_D = 2B(\hat{\mathbf{i}} + \hat{\mathbf{j}}) = 2\frac{\mu_0 I}{2\pi r} \cdot \sqrt{2}\hat{\mathbf{k}} = \frac{\sqrt{2}\mu_0 I}{\pi r}\hat{\mathbf{k}} = \frac{2\mu_0 I}{\pi a}\hat{\mathbf{k}} = 20\ \mu\text{T} \tag{13}$$

Problem 43. *Niobium metal becomes superconducting when cooled below 9K. Its superconductivity is destroyed when the surface B field exceeds $B_{max} = 0.100$ T. Determine the maximum current in a $d = 2.00$ mm diameter niobium wire can carry and remain superconducting, in the absence of any external B field.*

For long, cylindrical wires, the magnetic field a distance r from the center of the wire is

$$B = \frac{\mu_0 I}{2\pi r} \tag{14}$$

As long as you are outside the wire.

Therefore, the magnetic field at the surface is maximized when

$$B_{max} = \frac{\mu_0 I_{max}}{2\pi r} \tag{15}$$

$$I_{max} = (2\pi r B_{max})/\mu_0 = 500 \text{ A} \tag{16}$$

Problem 48. *In Bohr's 1913 model of the hydrogen atom, the electron is in a circular orbit of radius $r = 5.29 \cdot 10^{-11}$ m, and its speed is $v = 2.19 \cdot 10^6$ m/s. (a) What is the magnitude of the magnetic moment μ due to the electron's motion? (b) If the electron moves in a horizontal circle, counterclockwise as seen from above, what is the direction of μ?*

(a) The magnetic moment is defined on page 742 as

$$\mu = I\mathbf{A} \tag{17}$$

The area swept out by our electron is just

$$A = \pi r^2 \tag{18}$$

The current is the amount of charge circling the nucleus in a unit time. Because

$$\Delta x = v\Delta t \tag{19}$$

The time τ taken for an entire circuit is

$$\tau = \frac{\Delta x}{v} = \frac{2\pi r}{v} \tag{20}$$

The current is then given by

$$I = \frac{\Delta q}{\Delta t} = \frac{q_e v}{2\pi r} \tag{21}$$

Plugging I and A into our moment equation

$$\mu = \frac{q_e v}{2\pi r} \cdot \pi r^2 = (q_e v r)/2 = 9.27 \cdot 10^{-24} \text{ A m}^2 \tag{22}$$

The direction of the current is opposite the direction of the electron (because the electron has negative charge), so the direction of μ is down.

Problem 57. *A positive charge $q = 3.20 \cdot 10^{-19}$ C moves with a velocity $\mathbf{v} = (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}})$ m/s through a region where both a uniform magnetic field and a uniform electric field exist. (a) Calculate the total force F on the moving charge (in unit-vector notation), taking $\mathbf{B} = (2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + \hat{\mathbf{k}})$ T and $\mathbf{E} = (4\hat{\mathbf{i}} - \hat{\mathbf{j}} - 2\hat{\mathbf{k}})$ V/m. (b) What angle θ does the force vector $\mathbf{F}$ make with $\hat{\mathbf{i}}$?*

(a) From Chapter 19,

$$\mathbf{F}_E = q\mathbf{E} = q(4\hat{\mathbf{i}} - \hat{\mathbf{j}} - 2\hat{\mathbf{k}}) \text{ N/C} \tag{23}$$

From this chapter

$$\mathbf{F}_B = q\mathbf{v} \times \mathbf{B} = q\begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 2 & 3 & -1 \\ 2 & 4 & 1 \end{vmatrix} = q[(3+4)\hat{\mathbf{i}} - (2+2)\hat{\mathbf{j}} + (8-6)\hat{\mathbf{k}}] = q(7\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + 2\hat{\mathbf{k}}) \text{ N/C} \tag{24}$$

So the total force is given by

$$\mathbf{F} = \mathbf{F}_E + \mathbf{F}_B = q[(4+7)\hat{\mathbf{i}} + (-1-4)\hat{\mathbf{j}} + (-2+2)\hat{\mathbf{k}}] \text{ N/C} = q(11\hat{\mathbf{i}} - 5\hat{\mathbf{j}}) \text{ N/C} = (35.2\hat{\mathbf{i}} - 16.0\hat{\mathbf{j}}) \cdot 10^{-19} \text{ N} \tag{25}$$

(b)

$$\theta = \arctan\left(\frac{-5}{11}\right) = -24.4^\circ \tag{26}$$

Problem 58. *Protons having a kinetic energy of $K = 5.00$ MeV are moving in the $\hat{\mathbf{i}}$ direction and enter a magnetic field $B = 0.050\hat{\mathbf{k}}$ T directed out of the plane of the page and extending from $x = 0$ to $x = 1.00$ m as shown in Figure P22.58. (a) Calculate the y component of the protons' momentum as they leave the magnetic field. (b) Find the angle α between the initial velocity vector of the proton beam, and the velocity vector after the beam emerges from the field. Ignore relativistic effects and note that $1\ eV = 1.60 \cdot 10^{-19}$ J.*

(b) As in our cyclotron problem (Recitation 7, Problem 12), we know

$$F_c = m\frac{v^2}{r} = qvB \tag{27}$$

$$mv = qrB \tag{28}$$

And

$$K = \frac{1}{2}mv^2 \tag{29}$$

$$v = \sqrt{\frac{2K}{m}} = 30.9 \text{ Mm/s} \tag{30}$$

So the radius of the circular arc our protons make in the constant magnetic field region is

$$r = \frac{mv}{qB} = \frac{m}{qB}\sqrt{\frac{2K}{m}} = \frac{1}{qB}\sqrt{2Km} = 6.47 \text{ m} \tag{31}$$

Drawing out the center of the circle the beam would make and doing a bit of geometry, we see that

$$\alpha = \arcsin\left(\frac{\Delta x}{r}\right) = 8.90^\circ \tag{32}$$

(a) Because the *speed* of the particles doesn't change because of a magnetic field's perpendicular force, we can find the protons' speed in the y direction on exiting by

$$v_y = v\sin(\alpha) \tag{33}$$

So the y momentum is

$$p_y = mv_y = mv\sin(\alpha) = 0.155 \text{ kg m/s}^2 \tag{34}$$