

Recitation 7

Chapter 22

Problem 1. Determine the initial direction of the deflection of charged particles as they enter magnetic fields as shown in Figure P22.1.

Using our right hand rule for the cross product, and $\mathbf{F}_B = q\mathbf{v} \times \mathbf{B}$

- (a) Force is up.
- (b) Force is out of the page.
- (c) No force.
- (d) Force is into the page.

Problem 3. A proton travels with a speed of $v = 3.00 \cdot 10^6$ m/s at an angle of $\theta = 37.0^\circ$ with the direction of a magnetic field of $B = 0.300$ T in the $+y$ direction. What are (a) the magnitude of the magnetic force on the proton and (b) its acceleration?

We'll pick the $\hat{\mathbf{i}}$ direction so that \mathbf{v} has a positive x -component.

(a)

$$F_B = qvB \sin \theta = (1.60 \cdot 10^{-19} \text{ C}) \cdot (3.00 \cdot 10^6 \text{ m/s}) \cdot (0.300 \text{ T}) \cdot \sin 37.0^\circ = 8.67 \cdot 10^{-14} \text{ N} \quad (1)$$

and the direction of the force is in the $\hat{\mathbf{k}}$ direction.

(b) Using $\mathbf{F} = m\mathbf{a}$ we have

$$\mathbf{a} = \frac{\mathbf{F}}{m} = \frac{8.67 \cdot 10^{-14} \text{ N}}{1.67 \cdot 10^{-27} \text{ kg}} = 5.19 \cdot 10^{13} \text{ m/s}^2 \quad (2)$$

Problem 4. An electron is accelerated through $V = 2400$ V from rest and then enters a uniform $B = 1.70$ T magnetic field. What are (a) the maximum and (b) the minimum values of the magnetic force this charge can experience?

First we compute the electron's velocity v upon entering the field. Conserving energy

$$qV = \frac{1}{2}mv^2 \quad (3)$$

$$v = \sqrt{\frac{2qV}{m}} = 29.0 \text{ Mm/s} \quad (4)$$

The magnetic force is given by $\mathbf{F} = q\mathbf{v} \times \mathbf{B}$, so it is maximized when \mathbf{B} is perpendicular to \mathbf{v} , at which point

$$F = qvB = 7.90 \text{ pN} \quad (5)$$

The force is minimized then \mathbf{B} is parallel (or anti-parallel) to \mathbf{v} , at which point $F = 0$.

Problem 10. A velocity selector consists of electric and magnetic fields described by the expressions $\mathbf{E} = E\hat{\mathbf{k}}$ and $\mathbf{B} = B\hat{\mathbf{j}}$, with $B = 15.0$ mT. Find the value of E such that a $K = 750$ eV electron moving in the $\hat{\mathbf{i}}$ direction is undeflected.

The force from the magnetic field is in the $-\hat{\mathbf{k}}$ direction (right hand rule), so the sign of E must be negative (to push the electron in the $\hat{\mathbf{k}}$ direction).

The velocity of the electron is given by

$$K = \frac{1}{2}mv^2 \quad (6)$$

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2 \cdot (750 \text{ eV} \cdot 1.60 \cdot 10^{-19} \text{ J})}{9.11 \cdot 10^{-31} \text{ g}}} = 16.2 \text{ Mm/s} \quad (7)$$

Balancing the magnitudes of the two forces

$$F_e = qE = F_B = qvB \quad (8)$$

$$E = vB = 243 \text{ mV/m} \quad (9)$$

So $\mathbf{E} = -243\hat{\mathbf{k}}$ mV/m.

Problem 12. A cyclotron designed to accelerate protons has an outer radius of $R = 0.350$ m. The protons are emitted nearly at rest from a source at the center and are accelerated through $V = 600$ V each time they cross the gap between the dees. The dees are between the poles of an electromagnet where the field is $B = 0.800$ T. (a) Find the cyclotron frequency f . (b) Find the speed v_e at which the protons exit the cyclotron and (c) their kinetic energy K . (d) How many revolutions N does a proton make in the cyclotron? (e) For what time Δt interval does one proton accelerate?

(a) Protons with velocities v in a constant magnetic field will move in circles of radius r in the plane perpendicular to the magnetic field. The centerward acceleration is given by

$$F_c = m \frac{v^2}{r} = F_B = qvB \quad (10)$$

$$v = \frac{qrB}{m} \quad (11)$$

Their velocity can also be related to their period $T = 1/f$ by

$$v = \frac{dr}{dt} = \frac{2\pi r}{T} = 2\pi r f \quad (12)$$

So

$$2\pi r f = \frac{qrB}{m} \quad (13)$$

$$f = \frac{qB}{2\pi m} = 12.2 \text{ MHz} \quad (14)$$

This is the frequency of revolution for a proton *anywhere* inside the cyclotron.

(b) Using our $v(r)$ equation from (a) when $r = R$, we have

$$v_e = \frac{qRB}{m} = \frac{(1.60 \cdot 10^{-19} \text{ C}) \cdot (0.350 \text{ m}) \cdot (0.800 \text{ T})}{1.69 \cdot 10^{-27} \text{ kg}} = 26.8 \text{ Mm/s} \quad (15)$$

(c)

$$K = \frac{1}{2}mv^2 = \frac{(qrB)^2}{2m} = 6.01 \cdot 10^{-13} \text{ J} \quad (16)$$

(d) The kinetic energy K is built up from $2N$ passes through V (twice per revolution).

$$N = \frac{K}{2qV} = \frac{6.01 \cdot 10^{-13} \text{ J}}{2 \cdot (1.60 \cdot 10^{-19}) \cdot (600 \text{ V})} = 3130 \quad (17)$$

(e)

$$\Delta t = TN = N/f = 257 \mu\text{s} \quad (18)$$

Problem 15. A wire carries a steady current of $A = 2.40 \text{ A}$. A straight section of the wire is $l = 0.750 \text{ m}$ long and lies in the $\hat{\mathbf{i}}$ direction within a uniform magnetic field, $\mathbf{B} = 1.60\hat{\mathbf{k}} \text{ T}$. What is the magnetic force on the section of wire?

We can find the magnetic force on a wire using $\mathbf{F} = q\mathbf{v} \times \mathbf{B}$. Consider an infinitesimal bit of wire of length ds , a current $I = dq/dt$ means that dq will move through this bit of wire in time dt . So the force on the bit of wire is

$$d\mathbf{F} = dq \frac{d\mathbf{s}}{dt} \times \mathbf{B} = \frac{dq}{dt} d\mathbf{s} \times \mathbf{B} = I d\mathbf{s} \times \mathbf{B} \quad (19)$$

If the wire is straight, we can integrate easily to find the total force on the whole segment

$$\mathbf{F} = \int_0^l d\mathbf{F} = \int_0^l I d\mathbf{s} \times \mathbf{B} = IB \sin \theta \int_0^l ds = IlB \sin \theta = \mathbf{l} \times \mathbf{B} \quad (20)$$

Plugging in for our specific case we get a force in the $-\hat{\mathbf{j}}$ direction from the right-hand-rule, with a magnitude of

$$F = IlB = (2.40 \text{ A}) \cdot (0.750 \text{ m}) \cdot (1.60 \text{ T}) = 2.88 \text{ N} \quad (21)$$

So $\mathbf{F} = -2.88\hat{\mathbf{j}} \text{ N}$

Problem 21. A rectangular coil consists of $N = 100$ closely wrapped turns and has dimensions $a = 0.400 \text{ m}$ and $b = 0.300 \text{ m}$. The coil is hinged along the y axis, and its plane makes an angle $\theta = 30.0^\circ$ with the x axis (Fig. P22.21). What is the magnitude of the torque exerted on the coil by a uniform magnetic field $B = 0.800 \text{ T}$ directed along the x axis when the current is $I = 1.20 \text{ A}$ in the direction shown? What is the expected direction of motion of the coil?

Using our formula for force on a wire segment $\mathbf{F} = \mathbf{l} \times \mathbf{B}$, and recalling that torque is defined $\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$, we can use the right hand rule to find the direction of motion.

The torque from the portion of the coil lying on the y axis is zero, because the lever arm is zero (r in the torque equation). The torque from the top portion is also zero, because the force is in the $\hat{\mathbf{j}}$ direction and the coil is not free to rotate in that direction. Similarly the torque from the bottom portion is zero, because the force is in the $-\hat{\mathbf{j}}$ direction. All the torque comes from the force on the outer leg, giving a force in the $\hat{\mathbf{k}}$ direction. Thus, **we expect the angle θ to increase.**

To find the magnitude of the torque, we simply plug in

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F} = bF \cos \theta = b \cos \theta \cdot (NIaB) = IabB \cos \theta = 9.98 \text{ J} \quad (22)$$

Where we multiplied the force from a single wire by N because there are N wraps, and took $\cos \theta$ to get the perpendicular force because θ is the complement of the angle between r and F .