Recitation 5

Chapter 21

Problem 1. *In a particular cathode-ray tube, the measured beam current is* $I = 30.0\ \mu A$. *How many electrons strike the tube screen every* $\Delta t = 40.0\ s$

Current is defined as *charge passing through a given surface per unit time* or in SI units:

$$1\ \mathrm{A} = \frac{1\ \mathrm{C}}{1\ \mathrm{s}} \tag{1}$$

So

$$\Delta Q = I\Delta t = 1.20\ \mathrm{mC} \tag{2}$$

$$N_e = \frac{\Delta Q}{e} = 7.50 \cdot 10^{15} \tag{3}$$

Where $e = 1.60 \cdot 10^{-19}$ C is the charge on one electron.

Problem 4. *The quantity of charge q (in coulombs) that has passed through a surface of area* $A = 2.00\ cm^2$ *varies with time according to the equation* $q = 4t^3 + 5t + 6$, *where t is in seconds. (a) What is the instantaneous current across the surface at* $t_a = 1.00\ s$? *(b) What is the value of the current density?*

(a)

$$I(t) = \frac{dQ}{dt} = 12t^2 + 5 \tag{4}$$

$$I(t_a) = 17.0\ \mathrm{A} \tag{5}$$

(b)

$$j(t_a) = I(t_a)/A = 8.50\ \mathrm{A/cm^2} \tag{6}$$

Problem 14. *A toaster is rated at* $P = 600\ W$ *when connected to a* $V = 120\ V$ *source. What current I does the toaster carry, and what is its resistance R?*

(Assuming the voltage is DC). The power through a resistor is given by $P = IV$ so

$$I = \frac{P}{V} = \frac{600\ \mathrm{W}}{120\ \mathrm{V}} = 5\ \mathrm{A} \tag{7}$$

The voltage across a resistor is given by $V = IR$ so

$$R = \frac{V}{I} = \frac{120\ \mathrm{V}}{5\ \mathrm{A}} = 24\ \Omega \tag{8}$$

Problem 17. *Suppose a voltage surge produces* $V_s = 140\ V$ *for a moment. By what percentage p does the power output of a* $V = 120\ V$, $P = 100\ W$ *lightbulb increase? Assume the resistance does not change.*

The voltage across a resistor is

$$V = IR \tag{9}$$

So power absorbed by a resistor is

$$P = IV = \frac{V^2}{R} \tag{10}$$

And the fractional change in power f is given by

$$f = \frac{P_s}{P} = \frac{V_s^2/R}{V^2/R} = \left(\frac{V_s}{V}\right)^2 = 1.361 \tag{11}$$

So $p = 36.1\ \%$.

Problem 27. *(a) Find the equivalent resistance between points a and b in Figure P21.27. (b) A potential difference of* $V = 34.0\ V$ *is applied between points a and b. Calculate the current in each resistor.*

(Numbering from right to left and top to bottom, $R_1 = 4.00\ \Omega$, $R_2 = 7.00\ \Omega$, $R_3 = 10.0\ \Omega$, and $R_4 = 9.00\ \Omega$.)

(a) First, we calculate the equivalent resistance to the two resistors in parallel

$$R_p = \left(\frac{1}{R_2} + \frac{1}{R_1}\right)^{-1} = 4.12\ \Omega \tag{12}$$

Then we calculate the equivalent resistance of the three resistors in series

$$R_{ab} = R_1 + R_p + R_3 = 17.1\ \Omega \tag{13}$$

(b) Now applying $V = IR$ to the equivalent system

$$I_{ab} = I_1 = I_4 = I_p = \frac{V}{R_{ab}} = 1.99\ \text{A} \tag{14}$$

From which we can compute the voltage across the parallel resistors

$$V_p = I_p R_p = 8.18\ \text{V} \tag{15}$$

Giving us currents of

$$I_2 = \frac{V_p}{R_2} = 1.17\ \text{A} \tag{16}$$

$$I_3 = \frac{V_p}{R_3} = 0.818\ \text{A} \tag{17}$$

Problem 30. *Three $R = 100\ \Omega$ resistors are connected as shown in Figure P21.30. The maximum power that can safely be delivered to any one resistor is $P_{max} = 25.0$ W. (a) What is the maximum voltage that van be applies to the terminals a and b? (b) For the voltage determined in (a), what is the power delivered to each resistor? What is the total power delivered?*

(Numbering from right to left and top to bottom, R_1, R_2, and R_3.)

(a) The current through the entire setup I_{ab} all goes through R_1, so $I_{ab} = I_1$. Then it splits 50/50, so $I_{ab} = 2I_2 = 2I_3$. (R_1 and R_2 each get half the current going through R_1). Because it gets the most current, the maximum current I_{ab} is when the power P_1 absorbed by R_1 is P_{max}.

$$P_{max} = \frac{V_1^2}{R_1} \tag{18}$$

$$V_1 = \sqrt{R_1 P_{max}} = 50\ \text{V} \tag{19}$$

So $I_1 = I_{ab} = V_1/R_1 = .500$ A.

The equivalent resistance of the two parallel resistors is

$$R_p = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1} = 50\ \Omega \tag{20}$$

So the voltage drop over them is $V_p = I_{ab} R_p = 25.0$ V.

Adding the two voltages together

$$V_{ab} = V_1 + V_p = 75.0\ \text{V} \tag{21}$$

(b) The power absorbed by the other two resistors is then

$$P_2 = P_3 = I_2 V_p = 0.250\ \text{A} \cdot 25.0\ \text{V} = 6.25\ \text{W} \tag{22}$$

Problem 32. *Four resistors are connected to a battery as shown in Figure P21.32. The current in the battery is I, the battery emf is ε, and the resistor values are $R_1 = R$, $R_2 = 2R$, $R_3 = 4R$, and $R_4 = 3R$. (a) Rank the resistors according to the potential difference across them, form largest to smallest. Note any cases of equal potential difference. (b) Determine the potential difference across each resistor in terms of ε. (c) Rank the resistors according to the current in them, from largest to smallest. Note any cases of equal current. (d) Determine the current in each resistor in terms of I. (e) If R_3 is increased, what happens to the current in each of the resistors? (f) In the limit that $R \to \infty$, what are the new values of the current in each resistor in terms of I, the original current in the battery?*