

Recitation 4

Chapter 20

Problem 40. Two capacitors, $C_1 = 5.00 \mu\text{F}$ and $C_2 = 12.0 \mu\text{F}$, are connected in series, and the resulting combination is connected to a $\Delta V = 9.00 \text{ V}$ battery. Find (a) the equivalent capacitance of the combination, (b) the potential difference across each capacitor, and (c) the charge on each capacitor.

(a) The wire connecting the inner plates of C_1 and C_2 contains no net charge, so we know that any charge on the inner plate of C_1 must have come from the inner plate of C_2 . Because these charges are equal and opposite, the total charge Q on each capacitor separately is the same for both ($Q_1 = Q_2$). So using the definition of capacitance for both cases we have

$$\Delta V_1 = Q/C_1 \quad (1)$$

$$\Delta V_2 = Q/C_2 \quad (2)$$

$$\Delta V = \Delta V_1 + \Delta V_2 = Q \left(\frac{1}{C_1} + \frac{1}{C_2} \right) = \frac{Q}{C_{eq}} \quad (3)$$

So

$$C_{eq} = \left(\frac{1}{C_1} + \frac{1}{C_2} \right)^{-1} = \left(\frac{1}{5.00 \cdot 10^{-6} \text{ F}} + \frac{1}{12.0 \cdot 10^{-6} \text{ F}} \right)^{-1} = 3.53 \mu\text{F} \quad (4)$$

(c) Plugging back into equation 3 we have

$$Q = \Delta V \cdot C_{eq} = 3.53 \mu\text{F} \cdot 9.00 \text{ V} = 31.8 \mu\text{C} \quad (5)$$

(b) And plugging into equations 1 and 2 we have

$$\Delta V_1 = \frac{31.8 \cdot 10^{-6} \text{ C}}{5.00 \cdot 10^{-6} \text{ F}} = 6.35 \text{ V} \quad (6)$$

$$\Delta V_2 = \frac{31.8 \cdot 10^{-6} \text{ C}}{12.0 \cdot 10^{-6} \text{ F}} = 2.65 \text{ V} \quad (7)$$

(8)

Problem 43. Consider the circuit shown in Figure P20.43, where $C_1 = 6.00 \mu\text{F}$, $C_2 = 3.00 \mu\text{F}$, and $\Delta V = 20.0 \text{ V}$. Capacitor C_1 is first charged with Q_1 by the closing of switch S_1 . Switch S_1 is then opened, and the charged capacitor is connected to the uncharged capacitor by the closing of S_2 . Calculate Q_1 and the final charge on each capacitor (Q'_1 and Q'_2).

The first situation with S_1 closed and S_2 open is just a standard capacitor charging problem. Using the definition of capacitance

$$Q_1 = C_1 \Delta V = 6.00 \cdot 10^{-6} \text{ F} \cdot 20.0 \text{ V} = 120 \mu\text{C} \quad (9)$$

After disconnecting the battery and connecting the two capacitors, we have a net charge of Q_1 in the upper wire that we can distribute as we desire between C_1 and C_2 . Because charge is conserved, we know

$$Q_1 = Q'_1 + Q'_2 \quad (10)$$

We also know that at equilibrium the voltage across each capacitor must be equal (because if there was a voltage difference between the upper plates of the two capacitors, it would push current through the upper wire until the voltage difference disappeared, etc.). So

$$\Delta V'_1 = \frac{Q'_1}{C_1} = \Delta V'_2 = \frac{Q'_2}{C_2} \quad (11)$$

Now we have two equations relating our two unknowns Q'_1 and Q'_2 . Solving equation 11 for Q'_2 and plugging into equation 10 we get

$$Q'_2 = \frac{C_2}{C_1} Q'_1 \quad (12)$$

$$Q_1 = \left(1 + \frac{C_2}{C_1} \right) Q'_1 \quad (13)$$

$$Q'_1 = \frac{Q_1}{1 + C_2/C_1} = \frac{120 \mu\text{C}}{1.5} = 80 \mu\text{C} \quad (14)$$

$$Q'_2 = 0.5 \cdot 80 \mu\text{C} = 40 \mu\text{C} \quad (15)$$

Problem 47. (a) A $C = 4.00 \mu\text{F}$ capacitor is connected to a $\Delta V_a = 12.0 \text{ V}$ battery. How much energy U_a is stored in the capacitor? (b) If the capacitor had been connected to a $\Delta V_b = 6.00 \text{ V}$ battery, how much energy would have been stored?

Simply plugging into the formula for energy stored in a capacitor we have

$$U_a = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}(4.00 \cdot 10^{-6} \text{ F}) \cdot (12.0 \text{ V})^2 = 216 \mu\text{J} \quad (16)$$

$$U_b = \frac{1}{2}(4.00 \cdot 10^{-6} \text{ F}) \cdot (6.0 \text{ V})^2 = 54 \mu\text{J} \quad (17)$$

Problem 51. Show that the force between two plates of a parallel-plate capacitor each have an attractive force given by

$$F = \frac{Q^2}{2\epsilon_0 A} \quad (18)$$

The electric field generated by the plate A is given by $E_A = Q/2\epsilon_0 A$ (which we derived for P19.62, along with $\sigma = Q/A$). So the force on plate B due to plate A is given by

$$F = QE_A = \frac{Q^2}{2\epsilon_0 A} \quad (19)$$

Problem 54. (a) How much charge Q_c can be placed on a capacitor with air between the plates before it breaks down if the area of each plate is $A = 5.00 \text{ cm}^2$? (b) Find the maximum charge assuming polystyrene is used between the plates instead of air.

From Chapter 19, the voltage difference due to a constant electric field \mathbf{E} over a displacement \mathbf{d} is given by $\Delta V = \mathbf{E} \cdot \mathbf{d}$. So for two plates a distance d apart, the breakdown voltage is given by

$$V_c = E_c d \quad (20)$$

where E_c is the dielectric strength of the material.

The capacitance of a parallel-plate capacitor is given by

$$C = \frac{\kappa\epsilon_0 A}{d} \quad (21)$$

Combining these two formula with the definition of capacitance we have

$$E_c d = V = \frac{Q}{C} = \frac{Qd}{\kappa\epsilon_0 A} Q = \kappa E_c \epsilon_0 A \quad (22)$$

Looking up the values for air and polystyrene in Table 20.1 on page 699 of the text we see:

Name	Dielectric constant κ	Dielectric strength E_c
Air	1.00059	$3 \cdot 10^6 \text{ V/m}$
Polystyrene	2.56	$24 \cdot 10^6 \text{ V/m}$

So plugging into our formula for the charge

$$Q_a = 1.00 \cdot (3 \cdot 10^6 \text{ V/m}) \cdot (8.85 \cdot 10^{-12} \text{ C}^2/\text{N m}^2) \cdot 5 \cdot 10^{-4} \text{ m}^2 = 1.33 \cdot 10^{-8} \text{ C} \quad (23)$$

$$Q_b = 2.56 \cdot (24 \cdot 10^6 \text{ V/m}) \cdot (8.85 \cdot 10^{-12} \text{ C}^2/\text{N m}^2) \cdot 5 \cdot 10^{-4} \text{ m}^2 = 2.72 \cdot 10^{-7} \text{ C} \quad (24)$$

Problem 73. A parallel-plate capacitor is constructed using a dielectric material whose dielectric constant is $\kappa = 3.00$ and whose dielectric strength is $E_c = 2.00 \cdot 10^8 \text{ V/m}$. The desired capacitance is $C = 0.250 \mu\text{F}$, and the capacitor must withstand a maximum potential difference of $V_c = 4000 \text{ V}$. Find the minimum area A of the capacitor plates.

Using equation 20, we have

$$d \geq \frac{V_c}{E_c} \quad (25)$$

Where equality represents a breakdown at V_c and larger d give us more protection with larger breakdown voltages.

From equation 21 we have

$$A = \frac{dC}{\kappa\epsilon_0} \quad (26)$$

From which we can see that the smaller d is, the smaller A can be, and we pick $d = V_c/E_c$, the smallest possible value we can.

Then the smallest area is given by

$$A = \frac{V_c C}{E_c \kappa \epsilon_0} = \frac{(4000 \text{ V}) \cdot (0.25 \cdot 10^{-6} \text{ F})}{(2.00 \cdot 10^8 \text{ V/m}) \cdot 3.00 \cdot (8.85 \cdot 10^{-12} \text{ C}^2/\text{Nm}^2)} = 0.188 \text{ m}^2 \quad (27)$$