Recitation 2

Chapter 19

Problem 35. *A solid sphere of radius $R = 40.0$ cm has a total charge of $q = 26.0\ \mu C$ uniformly distributed throughout its volume. Calculate the magnitude E of the electric field (a) $r_a = 0$ cm, (b) $r_b = 10.0$ cm, (c) $r_c = 40.0$ cm, and (d) $r_d = 60.0$ cm from the center of the sphere.*

The charge distribution is symmetric under rotations and reflections about the center of the sphere , so the electric field must also be symmetric under rotations and reflections about the center of the sphere. So the electric field can only be a function of the radius $\mathbf{E}(r)$ (if it was a f'n of the angle, it wouldn't be symmetric under rotations), and it must be only in the radial direction $\mathbf{E}(r) = E(r)\hat{\mathbf{r}}$ (if it had non-radial components, it wouldn't be symmetric under reflections).

Because we have these insights from symmetry, we can use Gauss's Law to solve for $E(r)$

$$\oint \mathbf{E}(\mathbf{r}) \cdot d\mathbf{A} = \frac{q_{in}}{\varepsilon_0} \tag{1}$$

$$E(r) \oint \hat{\mathbf{r}} \cdot d\mathbf{A} = \frac{q_{in}}{\varepsilon_0} \tag{2}$$

because r is a constant over our surface of integration, $E(r)$ must also be constant, so we pull it out of the integral. We also note that $\hat{\mathbf{r}}$ is going to be perpendicular to our surface at every point on it, so

$$E(r) \oint dA = E(r)A = \frac{q_{in}}{\varepsilon_0} \tag{3}$$

$$E(r) 4\pi r^2 = \frac{q_{in}}{\varepsilon_0} \tag{4}$$

$$E(r) = \frac{q_{in}}{4\pi\varepsilon_0 r^2} \tag{5}$$

(If this is confusing, you can look at the first bit of the Gauss's law section 19.9 page 624 in the book for their derivation, and Example 19.9 on page 627 for their take on this problem.)

For $r \le R$ (points A, B, and C) we have

$$q_{in} = q\frac{4/3 \cdot \pi r^3}{4/3 \cdot \pi R^3} = q\left(\frac{r}{R}\right)^3 \tag{6}$$

so

$$E_{\le}(r) = \frac{qr^3/R^3}{4\pi\varepsilon_0 r^2} = \frac{qr}{4\pi\varepsilon_0 R^3} \tag{7}$$

$$E_a = 0 \qquad \text{because } r = 0 \tag{8}$$

$$E_b = \frac{26.0 \cdot 10^{-6}\ \mathrm{C} \cdot 0.100\ \mathrm{m}}{4\pi \cdot 8.854 \cdot 10^{-12}\ \mathrm{C^2/N{\cdot}m^2} \cdot (0.400\ \mathrm{m})^3} = 3.65 \cdot 10^5\ \mathrm{N/C} \tag{9}$$

And for $r \ge R$ (points C and D) we have $q_{in} = q$, so

$$E_{\ge}(r) = \frac{q}{4\pi\varepsilon_0 r^2} \tag{10}$$

$$E_c = 1.46 \cdot 10^6\ \mathrm{N/C} \tag{11}$$

$$E_d = 6.49 \cdot 10^5\ \mathrm{N/C} \tag{12}$$

Problem 38. *Consider a thin spherical shell of radius $R = 14.0$ cm with a total charge of $q = 32.0\ \mu C$ distributed uniformly on its surface. Find the electric field (a) $r = 10.0$ cm and (b) $r = 20.0$ cm from the center of the charge distribution.*

Again, the problem is symmetric under rotations and reflections about the center, so following the same reasoning as in Problem 35 we can use Equation 5.

(a) Inside the shell there is no charge ($q_{in} = 0$), so $E_a = 0$.

(b) Outside the shell we can use Equation 10

$$E_b = \frac{q}{4\pi\varepsilon_0 r_b^2} = \frac{32 \cdot 10^{-6}\ \mathrm{C}}{4\pi \cdot 8.853 \cdot 10^{-12}\ \mathrm{C^2/N{\cdot}m^2} \cdot (0.200\ \mathrm{m})^2} = 7.19 \cdot 10^6\ \mathrm{N/C} \tag{13}$$

Problem 40. *An insulating solid sphere of radius a has a uniform volume charge density ρ and carries a total positive charge Q. A spherical gaussian surface of radius r, which shares a common center with the insulating sphere, is inflated starting from $r = 0$. (a) Find an expression for the electric flux Φ_E passing through the surface of the gaussian sphere as a function of r for $r < a$. (b) Find an expression for the electric flux Φ_E for $r > a$. (c) Plot $\Phi_E(r)$.*

(a)

$$\Phi_E = \frac{q_{in}}{\varepsilon_0} = \frac{Q \cdot 4/3 \cdot \pi r^3}{\varepsilon_0 \cdot 4/3 \cdot \pi R^3} = \frac{Qr^3}{\varepsilon_0 R^3} \tag{14}$$

(b)

$$\Phi_E = \frac{q_{in}}{\varepsilon_0} = \frac{Q}{\varepsilon_0} \tag{15}$$

(c) Cubic increase followed continuously by a flat line.

Problem 57. *Two identical metallic blocks resting on a frictionless horizontal surface are connected by a light metallic spring having the spring constant $k = 100$ N/m and an unstretched length of $L_0 = 0.300$ m as shown in Figure P19.57a. A total charge of Q is slowly placed on the system, causing the spring to stretch to an equilibrium length of $L_1 = 0.400$ m as shown in Figure P19.57b. Determine the value of Q, assuming that all charge resides in the blocks and modeling the blocks as point charges.*

Looking at the right hand block (it doesn't matter which one you pick), we see that the only relevant forces are the attractive spring force, and the repulsive electrostatic force. Because the blocks are at equilibrium, these forces must cancel, so

$$E_s = k(L_1 - L_0) = E_E = k_e \frac{(Q/2)^2}{L_1^2} = k_e \left(\frac{Q}{2L_1}\right)^2 \tag{16}$$

$$Q = 2L_1 \sqrt{k(L_1 - L_0)/k_e} \tag{17}$$

$$= 2 \cdot 0.400 \text{ m} \cdot \sqrt{100 \text{ N/m} \cdot 0.100 \text{ m}/8.99 \cdot 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2} = 2.67 \cdot 10^{-5} \text{ C} \tag{18}$$

Problem 59. *Two small spheres of mass m are suspended from strings of length l that are connected at a common point. One sphere has charge Q, and the other has charge $2Q$. The strings make angles θ_1 and θ_2 with the vertical. (a) How are θ_1 and theta$_2$ related? (b) Assume that θ_1 and θ_2 are small. Show that the distance r between the spheres is given by*

$$r \approx \left(\frac{4k_e Q^2 l}{mg}\right)^{1/3} \tag{19}$$

(a) Assuming that the charges are not rotating about each other, the forces on each charge must cancel. The forces on each sphere are gravity $F_g = mg$, electrostatic $F_E = k_e 2Q^2/r^2$, and tension T. The tension will automatically handle canceling forces in the radial direction, so we need only consider the tangential direction. Let us assume that F_E is purely in the horizontal direction (see (Note)). Summing the tangential forces on the first sphere

$$0 = F_E \cos\theta_1 - F_g \sin\theta_1 \tag{20}$$

$$\tan\theta_1 = \frac{F_E}{F_g} \tag{21}$$

And on the second sphere $\tan\theta_2 = \frac{F_E}{F_g}$ so $\theta_1 = \theta_2 = \theta$.

(b)

$$r = 2l\sin\theta \approx 2l\tan\theta = 2l\frac{F_E}{F_g} = 2l\frac{k_e 2Q^2/r^2}{mg} \tag{22}$$

$$r \approx \left(\frac{4lk_e Q^2}{mg}\right)^{1/3} \tag{23}$$

(Note) Why $\mathbf{F}_E$ is horizontal.

Let q be the charge on the first mass and Q be the charge on the second. The force of 1 on 2 is given by $F_{12} = k_e qQ\hat{\mathbf{r}}_{12}/r^2$. This is identical to the force of 1 on 2 that we would get if we had put Q on 1 and q on 2 (let us say "the electric force does not care about which mass has which charge"). The only difference between the two masses is the charge, and the only effect of that difference (the electrostatic force) does not care about the difference, so the final situation must be symmetric ($\theta_1 = \theta_2$ [no calculation required :p] and $\mathbf{r}$ is horizontal). Because $\mathbf{F_E} \propto \hat{\mathbf{r}}_{12}$ it must also be horizontal.

Problem 62. *Two infinite, nonconducting sheets of charge are parallel to each other as shown in Figure P19.62. The sheet on the left has a uniform surface charge density σ, and the one on the right has a uniform charge density $-\sigma$. Calculate the electric field at points (a) to the left of, (b) in between, and (c) to the right of the two sheets.*

Let $\hat{\mathbf{i}}$ be the direction to the right perpendicular to the sheets. Because the problem has is symmetric to translations in the plane of the sheets and reflections through planes perpendicular to the sheets, the electric field must be of the form $\mathbf{E}(\mathbf{r}) = E(x)\hat{\mathbf{i}}$.

Using Gauss's law to find the electric field due to a single plate, we imagine a cylinder that extends through the plate a length L out either side. $\mathbf{E} = E\hat{\mathbf{i}}$, so no flux passes through the side walls of the cylinder. The single sheet is symmetric to reflection in it's plane, so (defining $x = 0$ to be the x value of the plane) $\mathbf{E}(x) = -\mathbf{E}(x)$ (positive charges are repelled from both sides). So, letting the area of a single end cap be A, the charge enclosed by the cylinder is σA and the flux through the end-caps of the cylinder is given by

$$\Phi_E = 2EA = \frac{q_{in}}{\varepsilon_0} = \frac{\sigma A}{\varepsilon_0} \tag{24}$$
$$E = \frac{\sigma}{2\varepsilon_0} \tag{25}$$

A constant! (See Example 19.12 on page 629 for the book's version)

(a) Using Equation 25 and superposition, we see

$$E_L = \frac{\sigma}{2\varepsilon_0} + \frac{-\sigma}{2\varepsilon_0} = 0 \tag{26}$$

(b)

$$\mathbf{E}_B = \frac{\sigma\hat{\mathbf{i}}}{2\varepsilon_0} + \frac{-\sigma\cdot(-\hat{\mathbf{i}})}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}\hat{\mathbf{i}} \tag{27}$$

(c) Identically to (a), $E_R = 0$.