Recitation 1

Chapter 19

Problem 3. *Nobel laureate Richard Feynman once said that if two persons stood at arm's length from each other and each person had $p = 1\%$ more electrons than protons, the force of repulsion between them would be enough to lift a "weight" equal to that of the entire Earth. Carry out an order of magnitude calculation to substantiate this assertion.*

Let $m = 70$ kg be the mass, $N = m/m_p$ be the number of protons, and $q = N \cdot p$ be the number of extra electrons in each person. Assume they are seperated by $r = 1$ m. The force of repulsion F is given by

$$F = k_e \frac{q^2}{r^2} = k_e \left(\frac{mpq_e}{m_p r}\right)^2 = 9.0 \cdot 10^9\ \text{N}{\cdot}\text{m}^2/\text{C}^2 \left(\frac{70\ \text{kg} \cdot 0.01 \cdot 1.6 \cdot 10^{-19}\ \text{C}}{1.7 \cdot 10^{-27}\ \text{kg} \cdot 1\ \text{m}}\right)^2 \approx 1 \cdot 10^{10} \left(\frac{1 \cdot 10^{-19}}{1 \cdot 10^{-27}}\right)^2 \text{N} = 1 \cdot 10^{26}\ \text{N} \tag{1}$$

And a "weight" the mass of the earth would be $F_g = Mg \approx 6 \cdot 10^{24}$ kg $< F$.

Problem 4. *Two protons in an atomic nucleus are typically seperated by a distance of $r = 2.00 \cdot 10^{-15}$ m. The electric repulsion force F between the protons is huge, but the attractive nuclear force is even stronger and keeps the nucleus from bursting apart. What is the magnitude of F?*

$$F = k_e \frac{q^2}{r^2} = 8.99 \cdot 10^9\ \text{N}{\cdot}\text{m}^2/\text{C}^2 \left(\frac{1.60 \cdot 10^{-19}\ \text{C}}{2.00 \cdot 10^{-15}\ \text{m}}\right)^2 = 57.7\ \text{N} \tag{2}$$

Problem 9. *In the Bohr theory of the hydrogen atom, an electron moves in a circular orbit about a proton, where the radius of the orbit is $r = 0.529 \cdot 10^{-10}$ m. (a) Find the magnitude of the electric force each exerts on the other. (b) If this force causes the centripetal acceleration of the electron, what is the speed of the electron?*

(a)

$$F = k_e \frac{q^2}{r^2} = 8.99 \cdot 10^9\ \text{N}{\cdot}\text{m}^2/\text{C}^2 \left(\frac{1.60 \cdot 10^{-19}\ \text{C}}{0.529 \cdot 10^{-10}\ \text{m}}\right)^2 = 8.24 \cdot 10^{-8}\ \text{N} \tag{3}$$

(b) Using $F_c = ma_c = mv^2/r$

$$v = \sqrt{\frac{Fr}{m}} = \sqrt{\frac{8.24 \cdot 10^{-8}\ \text{N} \cdot 0.529 \cdot 10^{-10}\ \text{m}}{9.11 \cdot 10^{-31}\ \text{kg}}} = 2.19 \cdot 10^6\ \text{m/s} \tag{4}$$

Problem 11. *In Figure P19.11, determine the point (other than infinity) at which the electric field is zero.*

First, we need a coordinate system. Let $q_1 = -2.50\ \mu$C be the origin ($x_1 = 0$), and $q_2 = 6.00\ \mu$C be at $x_2 = 1.00$ m.
The electric field of a finite number of point charge is given by (p. 612, 19.6)

$$\mathbf{E} = k_e \sum_i \frac{q_i}{r_i^2} \hat{\mathbf{r}}_i \tag{5}$$

A positive test charge placed between the two charges would be pulled to the left by q_1 and pushed to the left by q_2. A positive test charge placed to the right of q_2 would be pushed to the right by q_2 more strongly (because $q_2 > q_1$ and $r_2 < r_1$) than it would be pulled to the left by q_1. So the only place to look for equilibrium is to the left of q_1.

For any point off the x axis running through both charges, there would be some force moving the charge in the vertical y direction, so we only need to look at positions on the x axis for $x < 0$ (where $r_2 = r_1 + x_2$).

$$\mathbf{E} = k_e \left(-\frac{q_1}{r_1^2} - \frac{q_2}{(r_1 + x_2)^2}\right) = 0 \tag{6}$$

$$\frac{q_1}{r_1^2} = -\frac{q_2}{(r_1 + x_2)^2} \tag{7}$$

$$\frac{r_1 + x_2}{r_1} = \sqrt{\frac{-q_2}{q_1}} \tag{8}$$

$$1 + \frac{x_2}{r_1} = \sqrt{\frac{-q_2}{q_1}} \tag{9}$$

$$r_1 = \frac{x_2}{\sqrt{\frac{-q_2}{q_1}} - 1} = 1.82\ \text{m} \tag{10}$$

So $\mathbf{E} = 0$ at a point 1.82 m to the left of q_1.

Problem 15. *Four point charges are at the corners of a square of side a as shown in Figure P19.15. (a) Determine the magnitude and direction of the electric field at the location of charge q. (b) What is the resultant force on q?*

Let $\hat{\mathbf{i}}$ point to the right and $\hat{\mathbf{j}}$ point up.

$$\mathbf{E} = k_e \sum_i \frac{q_i}{r_i^2}\hat{\mathbf{r}}_i = k_e\left(\frac{2q}{a^2}\hat{\mathbf{i}} + \frac{3q}{(\sqrt{2}a)^2}\frac{\hat{\mathbf{i}}+\hat{\mathbf{j}}}{\sqrt{2}} + \frac{4q}{a^2}\hat{\mathbf{j}}\right) \tag{11}$$

$$= k_e\frac{q}{a^2}\left(2\hat{\mathbf{i}} + \frac{3}{2\sqrt{2}}(\hat{\mathbf{i}}+\hat{\mathbf{j}}) + 4\hat{\mathbf{j}}\right) \tag{12}$$

So the magnitude of $\mathbf{E}$ is given by

$$E = k_e\frac{q}{a^2}\sqrt{\left(2+\frac{3}{2\sqrt{2}}\right)^2 + \left(4+\frac{3}{2\sqrt{2}}\right)^2} = 5.91k_e\frac{q}{a^2} \tag{13}$$

And the direction θ (measured counter clockwise from $\hat{\mathbf{i}}$) of $\mathbf{E}$ is given by

$$\theta = \arctan\left(\frac{4+\frac{3}{2\sqrt{2}}}{2+\frac{3}{2\sqrt{2}}}\right) = 58.8^\circ \tag{14}$$

(b) $\mathbf{F} = q\mathbf{E}$ so the direction of $\mathbf{F}$ is the same as the direction of $\mathbf{E}$. The magnitude of $\mathbf{F}$ is given by $F = 5.91k_e q^2/a^2$

Problem 19. *A uniformly charged ring of radius $r = 10.0$ cm has a total charge of $q = 75.0$ μC. Find the electric field on the axis of the ring at (a) $x_a = 1.00$ cm, (b) $x_b = 5.00$ cm, (c) $x_c = 30.0$ cm, and (d) $x_d = 100$ cm from the center of the ring.*

From Example 19.5 (p. 616) we see the electric field along the axis ($\hat{\mathbf{i}}$) of a uniformly charged ring is given by

$$E = \frac{k_e x q}{(x^2+r^2)^{3/2}}\hat{\mathbf{i}} \tag{15}$$

So applying this to our 4 distances (rembering to convert the distances to meters), we have

$$E_a = 6.64 \cdot 10^6 \text{ N/C}\hat{\mathbf{i}} \tag{16}$$

$$E_b = 24.1 \cdot 10^6 \text{ N/C}\hat{\mathbf{i}} \tag{17}$$

$$E_c = 6.39 \cdot 10^6 \text{ N/C}\hat{\mathbf{i}} \tag{18}$$

$$E_d = 0.664 \cdot 10^6 \text{ N/C}\hat{\mathbf{i}} \tag{19}$$