Useful Equations

Chapter 19: Electric Forces and Electric Fields

Coulomb's law

$$\mathbf{F}_{12} = k_e \frac{q_1 q_2}{r^2} \hat{\mathbf{r}}_{12} \tag{1}$$

Where $k_e \approx 8.988 \cdot 10^9$ N·m^2/C^2 is the *Coulomb constant*, $\mathbf{F}_{12}$ is the force on q_1 due to q_2, and $\hat{\mathbf{r}}_{12}$ is a unit vector pointing from q_1 to q_2.

Electric field

$$\mathbf{E} \equiv \frac{\mathbf{F}_e}{q_0} \tag{2}$$

So for a point charge, the electric field at a point $\mathbf{r}$ is given by

$$\mathbf{E}(\mathbf{r}) = k_e \frac{q}{r^2} \hat{\mathbf{r}} \tag{3}$$

And for a group of charges, the electric field is given by

$$\mathbf{E}(\mathbf{r}) = k_e \int \frac{dq}{r^2} \hat{\mathbf{r}} \tag{4}$$

Electric flux

The electric flux is the amount of electric field "flowing" through a surface S:

$$\Phi_E \equiv \int_S \mathbf{E} \cdot d\mathbf{A} \tag{5}$$

When you know something about the symmetry of the problem, you can often use *Gauss's law*

$$\Phi_E = \oint \mathbf{E} \cdot d\mathbf{A} = \frac{q_{in}}{\varepsilon_0} \tag{6}$$

Where the $\oint$ symbol reminds us that the surface must be closed, q_{in} is the enclosed charge, and $\varepsilon_0 \approx 8.854 \cdot 10^{-12}$ C^2/N·m^2 is the *permittivity of free space*.

Chapter 20: Electric Potential and Capacitance

Potential

The change in potential ΔU of a charge q_0 moving from A to B in an electric field $\mathbf{E}(\mathbf{r})$ is given by

$$\Delta U = -q_0 \int_A^B \mathbf{E} \cdot d\mathbf{s} \tag{7}$$

Where $d\mathbf{s}$ is an infinitesimal displacement vector.

The electric potential is defined as

$$\Delta V = \frac{\Delta U}{q_0} = -\int_A^B \mathbf{E} \cdot d\mathbf{s} \tag{8}$$

Or, taking the derivative of both sides in the x direction

$$E_x = -\frac{dV}{dx} \tag{9}$$

With similar cases in the y and z directions. For those of you with vector calculus who recognize the gradient ∇ you can express the above formula as:

$$\mathbf{E} = -\nabla V \tag{10}$$

The voltage a distance r from a point charge q is given by

$$V = -\int_A^B \frac{k_e q}{r^2} dr = k_e \frac{q}{r} \tag{11}$$

Which we can combine with the definition of the electric potential to find the potential energy of two point charges seperated by a distance r_{12}:

$$U = q_1 V = k_e \frac{q_1 q_2}{r_{12}} \tag{12}$$

Just as we could integrate over a charge distribution to find the electric field at a given point, we can find electric potential with

$$V = k_e \int \frac{dq}{r} \tag{13}$$

Which is usually an easier integral because the integrand is a *scalar*.

Capacitance

The capacitance C of a pair of conductors (plates, other shapes...) is defined as

$$C \equiv \frac{Q}{\Delta V} \tag{14}$$

Where Q is the charge on one of two oppositely charged conductors, and ΔV is the voltage difference between the conductors.

The capacitance of a capacitor consisting of two parallel plates of area A seperated by a distance d is given by

$$C = \frac{\varepsilon_0 A}{d} \tag{15}$$

where ε_0 is the permittivity of free space. (Which comes from using the definition of capacitance, and the electric field between two charged plates that we calculated in recitation 2, problem P19.62.)

The equivalent capacitance of several capacitors in parallel is

$$C_{eq} = C_1 + C_2 + C_3 + \cdots \tag{16}$$

and in series is

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots \tag{17}$$

The electric potential energy U stored in a capacitor is

$$U = \frac{1}{2} C (\Delta V)^2 \tag{18}$$