Homework 5

Chapter 21

Problem 25. *A battery has an emf of $\varepsilon = 15.0$ V. THe terminal voltage of the battery is $V_t = 11.6$ V when it is delivering $P = 20.0$ W of power to an external load resistor R. (a) What is the value of R? (b) What is the internal resistance r of the battery?*

(a) Using the power absorbed by R

$$P = IV_t = \frac{V_t^2}{R} \tag{1}$$

$$R = \frac{V_t^2}{P} = \frac{(11.6\text{ V})^2}{20.0\text{ W}} = 6.75\Omega \tag{2}$$

(b) The current through the entire ciruit is given by

$$I = \frac{V_t}{R} = \frac{P}{V_t} = 1.72\text{ A} \tag{3}$$

So the internal resistance is given by

$$\varepsilon - V_t = Ir \tag{4}$$

$$r = \frac{\varepsilon - V_t}{I} = \frac{3.4\text{ V}}{1.72\text{ A}} = 1.79\Omega \tag{5}$$

Problem 29. *Consider the circuit shown in Figure P21.29. Find (a) the current in the $R_1 = 20\Omega$ resistor and (b) the potential difference between points a and b.*

Label the voltage $V = 25.0$ V and the resistances (clockwise from b) $R_1 = 20.0\Omega$, $R_2 = 5.00\Omega$, $R_3 = 10.0\Omega$, $R_4 = 10.0\Omega$, and $R_5 = 5.00\Omega$.

Computing some equivalent resistance of R_1 and R_2 in series we have

$$R_s = R_1 + R_2 = 25.0\Omega \tag{6}$$

Computing the equivalent resistance of R_s, R_4, and R_5 in parallel we have

$$R_p = \left(\frac{1}{R_4} + \frac{1}{R_5} + \frac{1}{R_s}\right)^{-1} = 2.94\Omega \tag{7}$$

And the equivalent resistance of the entire setup is

$$R_e = R_p + R_3 = 12.94\Omega \tag{8}$$

The total current is then (from Ohm's law)

$$I_e = \frac{V}{R_e} = 1.93\text{ A} \tag{9}$$

And the voltage from a to b is

$$V_{ab} = I_e R_p = 5.68\text{ V} \tag{10}$$

Which is what they were looking for in (b).

The current through the branch with R_1 and R_2 is then

$$I_s = \frac{V_{ab}}{R_s} = 227\text{ mA} \tag{11}$$

Which is what they were looking for in (a).

Problem 31. *Calculate the power delivered to each resistor in th circuit shown in Figure P21.31.*

Label the voltage $V = 18.0$ V and the resistors (starting in the upper left) $R_1 = 2.00\Omega$, $R_2 = 3.00\Omega$, $R_3 = 1.00\Omega$, and $R_4 = 4.00\Omega$. To find the total current through the circuit, we compute its equivalent resistance. First for the two resistors in parallel

$$R_p = \left(\frac{1}{R_2} + \frac{1}{R_3}\right) - 1 = 0.750\Omega \tag{12}$$

And then for the complete circuit

$$R_c = R_1 + R_p + R_4 = 6.75\Omega \tag{13}$$

Using Ohm's law to calculate the total current $I_c = I_1 = I_4$ we have

$$I_c = \frac{V}{R_c} = 18/6.75 = 2.67 \text{ A} \tag{14}$$

And the powers dissipated through R_1 and R_4 are

$$P_1 = I_1 V_1 = I_c^2 R_1 = 14.2 \text{ W} \tag{15}$$

$$P_4 = I_c^2 R_4 = 28.4 \text{ W} \tag{16}$$

The voltage across R_p is given by

$$V_p = I_c R_p = 2 \text{ V} \tag{17}$$

And the powers dissipated through R_2 and R_3 are

$$P_2 = I_2 V_2 = \frac{V_p^2}{R_2} = 1.33 \text{ W} \tag{18}$$

$$P_3 = \frac{V_p^2}{R_3} = 4 \text{ W} \tag{19}$$