# Homework 2

Chapter 19

**Problem 31.** *A $d = 40.0$ cm diameter loop is rotated in a uniform electric field until the position of maximum electric flux is found. The flux in this position is measured to be $\Phi_E = 5.20 \cdot 10^5$ N·m$^2$/C. What is the magnitude of the electric field?*

$$\Phi_E = EA \tag{1}$$

$$E = \frac{\Phi_E}{A} = \frac{\Phi_E}{\pi(d/2)^2} = \frac{5.20 \cdot 10^5 \text{ N·m}^2\text{/C}}{\pi \cdot (0.200 \text{ m})^2} = 4.14 \cdot 10^6 \text{ N/C} \tag{2}$$

**Problem 36.** *An $m = 10.0$ g piece of Styrofoam carries a net charge of $q = -0.700$ μC and floats above the center of a large horizontal sheet of plastic that has a uniform charge density σ on it's surface. Find σ.*

Because the Styrofoam is floating in equilibrium, the sum of forces in the vertical direction must be zero. So

$$F_g = mg = F_E = qE = q\frac{\sigma}{2\varepsilon_0} \tag{3}$$

$$\sigma = \frac{2\varepsilon_0 mg}{q} = \frac{2 \cdot 8.54 \cdot 10^{-12} \text{ C}^2\text{/N·m}^2 \cdot 0.0100 \text{ kg} \cdot 9.80 \text{ m/s}^2}{-0.700 \cdot 10^{-6} \text{ C}} = 2.39 \cdot 10^{-6} \text{ C/m}^2 \tag{4}$$

**Problem 55.** *Four identical point charges ($q = +10.0$ μC) are located on the corners of a rectangle as shown in Figure P19.55. The dimensions of the rectangle are $L = 60.0$ cm and $W = 15.0$ cm. Calculate the magnitude and direction of the resultant electric force exerted on the charge at the lower left corner by the other three charges.*

This is just a jazzed up version of Problem 15 from recitation. The unit vector $\hat{\mathbf{r}}$ diagonally across from the upper right is given by

$$\hat{\mathbf{r}} = \cos\theta\hat{\mathbf{i}} + \sin\theta\hat{\mathbf{j}} \tag{5}$$

$$\theta = \arctan W/L + 180^\circ = 194^\circ \tag{6}$$

$$\cos\theta = -0.970 \tag{7}$$

$$\sin\theta = -0.243 \tag{8}$$

So the electric field in the lower left corner is given by

$$\mathbf{E} = k_e \sum_i \frac{q_i}{r_i^2}\hat{\mathbf{r}}_i = k_e \left( \frac{q}{L^2}(-\hat{\mathbf{i}}) + \frac{q}{(L^2+W^2)}(\cos\theta\hat{\mathbf{i}} + \sin\theta\hat{\mathbf{j}}) + \frac{q}{W^2}(-\hat{\mathbf{j}}) \right) \tag{9}$$

$$= -k_e q \left[ \left( \frac{1}{L^2} - \frac{\cos\theta}{L^2+W^2} \right)\hat{\mathbf{i}} + \left( \frac{1}{W^2} - \frac{\sin\theta}{L^2+W^2} \right)\hat{\mathbf{j}} \right] \tag{10}$$

So the magnitude of $\mathbf{E}$ is given by

$$E = k_e q \sqrt{\left( L^{-2} - \frac{\cos\theta}{L^2+W^2} \right)^2 + \left( W^{-2} - \frac{\sin\theta}{L^2+W^2} \right)^2} = 4.08 \cdot 10^6 \text{ N/C} \tag{11}$$

(Remembering to convert $L$ and $W$ to meters.) And the direction θ (measured counter clockwise from $\hat{\mathbf{i}}$) of $\mathbf{E}$ is given by

$$\theta = \arctan\left( \frac{-W^{-2} + \frac{\sin\theta}{L^2+W^2}}{-L^{-2} + \frac{\cos\theta}{L^2+W^2}} \right) + 180^\circ = 263^\circ \tag{12}$$

Where the $+180^\circ$ is because the tangent has a period of $180^\circ$, and the angle we want is in the backside $180^\circ$.

$\mathbf{F} = q\mathbf{E}$ so the direction of $\mathbf{F}$ is the same as the direction of $\mathbf{E}$. The magnitude of $\mathbf{F}$ is given by

$$F = 10.0 \cdot 10^{-6} \text{ C} \cdot 4.08 \cdot 10^6 \text{ N/C} = 40.8 \text{ N} \tag{13}$$