Homework 1

Chapter 19

Problem 7. *Two identical conducting small spheres are placed with their centers $r = 0.300$ m apart. One is given a charge of $q_1 = 12.0$ nC and the other a charge of $q_2 = -18.0$ nC. (a) Find the electric force exerted by one sphere on the other. (b) Next, the spheres are connected by a conducting wire. Find the electric force between the two after they have come to equilibrium.*

(a)

$$F = k_e \frac{q_1 q_2}{r^2} = 8.99 \cdot 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2 \frac{12.0 \cdot 10^{-9} \text{ C} \cdot (-18.0) \cdot 10^{-9} \text{ C}}{(0.300 \text{ m})^2} = -2.16 \cdot 10^{-5} \text{ N} \tag{1}$$

And the force is towards the other sphere for each sphere because opposites attract.

(b) The total charge on the both spheres is $Q = q_1 + q_2 = -6.0$ nC. The spheres are identical, so at equilibrium, there will be $Q/2 = -3.0$ nC on each sphere. The repulsive (since now they have the same charge sign) force between them is given by

$$F = k_e \frac{(Q/2)^2}{r^2} = 8.99 \cdot 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2 \left(\frac{-3.0 \cdot 10^{-9} \text{ C}}{0.300 \text{ m}} \right)^2 = 8.99 \cdot 10^{-7} \text{ N} \tag{2}$$

Problem 13. *Three point charges are arranged as shown in Figure P19.13.*
(a) Find the vector electric field **E** *that q_2 and q_3 together create at the origin. (b) Find the vector force* **F** *on q_1.*

Name	Charge (nC)	x (m)	y (m)
q_1	5.00	0	0
q_2	6.00	0.300	0
q_3	-3.00	0	-0.100

(a)

$$\mathbf{E} = k_e \sum_i \frac{q_i}{r_i^2} \hat{\mathbf{r}}_i = k_e \left[\frac{q_2}{x_2^2}(-\hat{\mathbf{i}}) + \frac{q_3}{y_3^2}\hat{\mathbf{j}} \right] = 8.99 \cdot 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2 \left(\frac{-6.00\hat{\mathbf{i}}}{0.300^2} - \frac{3.00\hat{\mathbf{j}}}{0.100^2} \right) \cdot 10^{-9} C/m^2 = \left(-0.599\hat{\mathbf{i}} - 2.70\hat{\mathbf{j}} \right) \text{ kN/C} \tag{3}$$

(b)

$$\mathbf{F} = q_1 \mathbf{E} = \left(-3.00\hat{\mathbf{i}} - 13.5\hat{\mathbf{j}} \right) \mu\text{N} \tag{4}$$

Problem 16. *Consider the electric dipole shown in Figure P19.16. Show that the electric field at a distant point on the $+x$ axis is $E_x \approx 4k_e qa/x^3$.*

Let us assume the point in question has a positive x value (just reverse the sign if $x < 0$).

$$\mathbf{E} = k_e \sum_i \frac{q_i}{r_i^2} \hat{\mathbf{r}}_i = k_e \left[\frac{q}{(x-a)^2}\hat{\mathbf{i}} + \frac{-q}{(x+a)^2}\hat{\mathbf{i}} \right] \tag{5}$$

For $|x| \gg |c|$,

$$(x+c)^n = x^n \left(1 + \frac{c}{x}\right)^n = x^n \left[1 + n\frac{c}{x} + \frac{n(n-1)}{2} \cdot \left(\frac{c}{x}\right)^2 + \dots \right] \approx x^n (1 + n\frac{c}{x}) \tag{6}$$

Because $(c/x)^2$ is very, very small. (What we are doing, is just Taylor expanding $(x+c)^n$ as a function of c/x, and keeping only the first two terms.) In our case, $n = -2$ and $c = \mp a$

$$\mathbf{E} = k_e \left[\frac{q}{x^2} \left(1 - 2\frac{-a}{x} \right) + \frac{-q}{x^2} \left(1 - 2\frac{a}{x} \right) \right] \hat{\mathbf{i}} = k_e \frac{q}{x^2} \left(1 + 2\frac{a}{x} - 1 + 2\frac{a}{x} \right) \hat{\mathbf{i}} = \frac{4k_e qa}{x^3} \hat{\mathbf{i}} \tag{7}$$