

Recitation 8

Chapter 8

Problem 5. Two blocks with masses M and $3M$ are played on a horizontal, frictionless surface. A light spring is attached to one of them, and the blocks are pushed to gether with the spring between them (Fig. P8.5). A cord initially holding the blocks together is burned; after this, the block of mass $3M$ moves to the right with a speed of $v_B = 2.00$ m/s. (a) What is the speed v_S of the block of mass M ? (b) Find the original elastic potential energy U_s in the spring, taking $M = 0.350$ kg

(a) Conserving momentum

$$P_i = 0 = P_f = P_B - P_S = 3Mv_B - Mv_S \quad (1)$$

$$v_S = 3v_B = 3 \cdot 2.00 \text{ m/s} = \mathbf{6.00 \text{ m/s}} \quad (2)$$

(b) Conserving energy

$$E_i = U_s = E_f = K_B + K_S = \frac{1}{2}3Mv_B^2 + \frac{1}{2}Mv_S^2 \quad (3)$$

$$U_s = \frac{1}{2}3Mv_B^2 + \frac{1}{2}M(3v_B)^2 = \frac{1}{2}Mv_B^2(3+9) = 6Mv_B^2 = 5 \cdot 0.350 \text{ kg} \cdot (2.00 \text{ m/s})^2 = \mathbf{8.40 \text{ J}} \quad (4)$$

Problem 17. Suppose a truck and car with initial speeds of $v_i = 8.00$ m/s collide in a perfectly inelastic head on collision. Each driver has a mass of $m = 80.0$ kg. Including the drivers, the total vehicle masses are $m_c = 800$ kg for the car and $m_t = 4000$ kg for the truck. If the collision time is $\Delta t = 0.120$ s, what force does the seat belt exert on each driver.

To find the final velocity v_f of the crumpled mass, we conserve momentum.

$$P_i = m_t v_i - m_c v_i = (m_t - m_c)v_i = P_f = (m_t + m_c)v_f \quad (5)$$

$$v_f = v_i \frac{m_t - m_c}{m_t + m_c} = 8.00 \text{ m/s} \frac{4000 \text{ kg} - 800 \text{ kg}}{4800 \text{ kg}} = 5.333 \dots \text{ m/s} \quad (6)$$

In the same direction the truck was initially going. And the average force F on each driver is then given by

$$F = ma = m \frac{\Delta v}{\Delta t} = m \frac{v_f - v_i}{\Delta t} \quad (7)$$

$$F_t = m \frac{v_f - v_i}{\Delta t} = 80.0 \text{ kg} \frac{5.333 \text{ m/s} - 8.00 \text{ m/s}}{0.120 \text{ s}} = \mathbf{-1780 \text{ N}} \quad (8)$$

$$F_c = m \frac{v_f + v_i}{\Delta t} = 80.0 \text{ kg} \frac{5.333 \text{ m/s} + 8.00 \text{ m/s}}{0.120 \text{ s}} = \mathbf{8890 \text{ N}} \quad (9)$$

Problem 18. As show in Fig. P8.18, a bullet of mass m and speed v passes completely through a pendulum bob of mass M . The bullet emerges with a speed $v_f = v/2$. The pendulum bob is suspended by a stiff rod of length l and a negligible mass. What is the minimum value of v such that the pendulum bob will barely swing through a complete vertical circle?

Let us break the problem up into two steps: the collision where we'll conserve momentum, and the pendulum swinging upside down where we'll conserve energy. Call the point before the collision A , the point just after the collision before the bob has started to swing B , and the point where the pendulum is completely inverted C . We just need to give the bob enough energy that it has no speed at C (just barely coasting through), so conserving energy back to B

$$E_C = Mg(2l) = E_B = \frac{1}{2}Mv_B^2 \quad (10)$$

$$v_B^2 = 4gl \quad (11)$$

$$v_B = 2\sqrt{gl} \quad (12)$$

Where we've left out the kinetic energy of the bullet since it doesn't change from B to C

Now conserving momentum back to A

$$P_B = Mv_B + m(v/2) = P_A = mv \quad (13)$$

$$v = \frac{2Mv_B}{m} = \frac{4M}{m} \sqrt{gl} \quad (14)$$

Problem 25. Two shuffleboard disks of equal mass, one orange and the other yellow, are involved in an elastic, glancing collision. The yellow disk is initially at rest and is struck by the orange disk moving with a speed v_i . After the collision, the orange disk moves along a direction that makes an angle θ with its initial direction of motion. The velocities of the two disks are perpendicular after the collision. Determine the final speed of each disk.

Let the final speed of the orange disk be v_o , the final speed of the yellow disk be v_y , and m be the mass of one disk. Calling the initial direction of the orange disk \hat{i} , and the direction perpendicular to that \hat{j} (such that the final direction of \mathbf{v}_o has positive components in both directions), we see

$$v_{o\hat{i}} = v_o \cos \theta \quad (15)$$

$$v_{o\hat{j}} = v_o \sin \theta \quad (16)$$

For the orange puck, and that since the motion of the yellow is perpendicular to the orange, the angle between the final motion of the yellow and the $-\hat{j}$ direction is also θ , so

$$v_{y\hat{i}} = v_y \sin \theta \quad (17)$$

$$v_{y\hat{j}} = -v_y \cos \theta \quad (18)$$

Conserving momentum in both directions we have

$$P_{i\hat{j}} = 0 = P_{f\hat{j}} = mv_{y\hat{j}} + mv_{o\hat{j}} = mv_o \sin \theta - mv_y \cos \theta \quad (19)$$

$$v_y = v_o \frac{\sin \theta}{\cos \theta} \quad (20)$$

$$P_{i\hat{i}} = mv_i = P_{f\hat{i}} = mv_{y\hat{i}} + mv_{o\hat{i}} = mv_o \cos \theta + mv_y \sin \theta \quad (21)$$

$$v_i \cos \theta = v_o \cos^2 \theta + \left(v_o \frac{\sin \theta}{\cos \theta} \right) \sin \theta \cos \theta = v_o \cos^2 \theta + v_o \sin^2 \theta = v_o \quad (22)$$

Because

$$\sin^2 \theta + \cos^2 \theta = 1 \quad (23)$$

So

$$v_o = v_i \cos \theta \quad (24)$$

$$v_y = v_o \frac{\sin \theta}{\cos \theta} = v_i \sin \theta \quad (25)$$

Problem 26. Two automobiles of equal mass approach an intersection. One vehicle is traveling with a velocity $v_1 = 13.0$ m/s towards the east (\hat{i}), and the other is traveling north (\hat{j}) with a speed v_2 . Neither driver sees the other. The vehicles collide in the intersection and stick together, leaving parallel skid marks at an angle of $\theta = 55.0^\circ$ north of east. The speed limit for both roads is 35 mph, and the driver of the northward-moving vehicle claims that he was within the speed limit when the collision occurred. Is he telling the truth?

Let m be the mass of one car and \mathbf{v}_f be the final velocity of the wreck. Conserving momentum in both directions

$$P_{i\hat{i}} = mv_1 = P_{f\hat{i}} = (m+m)v_f \cos \theta \quad (26)$$

$$P_{i\hat{j}} = mv_2 = P_{f\hat{j}} = (m+m)v_f \sin \theta \quad (27)$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{mv_2}{mv_1} \quad (28)$$

$$v_2 = v_1 \tan \theta = 13.0 \text{ m/s} \tan 55.0^\circ \cdot \frac{1 \text{ mi}}{1609 \text{ m}} \cdot \frac{3600 \text{ s}}{1 \text{ h}} = 41.6 \text{ mph} \quad (29)$$

So he was speeding.

Problem 43. A rocket for use in deep space is to be capable of boosting a total load (payload plus rocket frame and engine) of $M_f = 3.00$ metric tons to a speed of $v_f = 10.0$ km/s. (a) It has an engine and fuel designed to produce an exhaust speed of $v_{ea} = 2.000$ km/s. How much fuel plus oxidizer is required? (b) If a different fuel and engine design could give an exhaust speed of $v_{eb} = 5.000$ km/s, what amount of fuel and oxidizer would be required for the same task?

(a) Starting with equation 8.43 from page 248, and letting $M_e = M_i - M_f$ be the mass of the fuel and oxidizer

$$v_f - v_i = v_e \ln \left(\frac{M_i}{M_f} \right) \quad (30)$$

$$M_i = M_f \exp \frac{v_f - v_i}{v_e} \quad (31)$$

$$M_e = M_f \left(\exp \frac{v_f - v_i}{v_e} - 1 \right) \quad (32)$$

$$= 3.00 \text{ metric tons} \left(\exp \frac{10}{2} - 1 \right) = 442 \text{ metric tons} \quad (33)$$

(b) Using eqn. 32 with our new exhaust velocity,

$$M_e = 3.00 \text{ metric tons} \left(\exp \frac{10}{5} - 1 \right) = 19.2 \text{ metric tons} \quad (34)$$

Problem 45. An orbiting spacecraft is described not as a “zero-g” but rather as a “microgravity” environment for its occupants and for onboard experiments. Astronauts experience slight lurches due to the motions of the equipment and other astronauts and as a result of venting of materials from the craft. Assume that an $M_i = 3500$ kg spacecraft undergoes an acceleration of $a = 2.50 \mu g = 2.45 \cdot 10^{-5} \text{ m/s}^2$ due to a leak from one of its hydraulic control systems. The fluid is known to escape with a speed of 70.0 m/s into the vacuum of space. How much fluid will be lost in $\Delta t = 1.00 \text{ h}$ if the leak is not stopped.

If the acceleration of the spaceship remains constant, and the rate of fluid escape remains constant, the mass of escaping fluid must be much less than the mass of the spaceship. Conserving momentum according to the conservation of momentum equation 8.42 in the text,

$$Mdv = -v_e dM \quad (35)$$

$$dM = -M \frac{dv}{v_e} = -M \frac{a\Delta t}{v_e} = -3500 \text{ kg} \frac{2.45 \cdot 10^{-5} \text{ m/s}^2 \cdot 3600 \text{ s}}{70.0 \text{ m/s}} = -4.41 \text{ kg} \quad (36)$$

Problem 48. A bullet of mass m is fired horizontally into a block of mass M initially at rest at the edge of a frictionless table of height h (Fig. P8.48). The bullet remains in the block, and after impact the block lands a distance d from the bottom of the table. Determine the initial speed of the bullet.

Breaking the problem up into two parts (like problem 18), call the point before the collision A , the point just after the collision B and the point when the block-bullet hits the floor C .

From B to C is a standard projectile motion problem, which we'll solve for the horizontal velocity v_B of the block-bullet at point B . Because v_B is purely horizontal (the \hat{i} direction), we'll use the vertical (\hat{j}) direction to find the time it took the ball to fall.

$$y_f = -h = \frac{1}{2}at^2 + v_{y0}t + y_0 = \frac{-g}{2}t^2 \quad (37)$$

$$t = \sqrt{\frac{2h}{g}} \quad (38)$$

$$x_f = d = v_{Bt} + x_0 = v_B t \quad (39)$$

$$v_B = \frac{d}{t} = d \sqrt{\frac{g}{2h}} \quad (40)$$

$$(41)$$

Now conserving momentum back to A

$$P_B = (m + M)v_B = P_A = mv \quad (42)$$

$$v = \frac{m + M}{m} v_B = \frac{m + M}{m} d \sqrt{\frac{g}{2h}} \quad (43)$$