Recitation 7

Problem 2. A $F_g = 400$ N child is in a swing attached to $r = 2.00$ m ropes. Find the gravitional potential energy U_g of the child-Earth system relative to the child's lowest position when (a) the ropes are horizontal, (b) the ropes make a $\theta = 30.0°$ angle with the vertical, and (c) the child is at the bottom of the cirvular arc.

U_g is given by

$$U_g = mgh = F_g h \tag{1}$$

Therefore, we need to determine the vertical distance between the child's location for a given part of the question and the child's lowest position.

(a) The child is one radius above the lowest position, so

$$U_{gA} = F_g r = 400\text{ N} \cdot 2.00\text{ m} = 800\text{ J} \tag{2}$$

(b) The child has height $h_b = (1 - \cos\theta)r$, so

$$U_{gB} = F_g(1-\cos\theta)r = U_{gA} \cdot (1-\cos\theta) = 800\text{ J} \cdot (1 - \cos 30.0°) = 107\text{ J} \tag{3}$$

(c) The child has a height of 0, so $U_{gC} = 0$ J.

Problem 10. A particle of mass $m = 5.00$ kg is released from point A and slides on the frictionless track shown in Figure P7.10. Determine (a) the particle's speed at points B and C and (b) the net work done by the gravitaional force as the particle moves from A to C.

Reading heights from the figure,

Point	Height	Energy
A	5.00 m	U_{gA}
B	3.20 m	$U_{gB} + K_B$
C	2.00 m	$U_{gC} + K_C$

Where the energies are simply the sum of the particle's kinetic and gravitational potential energies. The particle has no kinetic energy at A because is is released from rest.

The track is frictionless so there are no non-conservative forces acting on the particle. Therefore the particle's energy is conserved.

(a) Conserving energy, we have

$$E_A = U_{gA} = E_B = U_{gB} + K_B \tag{4}$$
$$K_B = U_{gA} - U_{gB} \tag{5}$$
$$\frac{1}{2}mv_B^2 = mgh_A - mgh_B = -mg\Delta h_{AB} \tag{6}$$
$$v_B = \sqrt{-2g\Delta_{hAB}} = \sqrt{-2 \cdot 9.8\text{ m/s}^2 \cdot (3.20 - 5.00)\text{ m}} = 5.94\text{ m/s} \tag{7}$$

And applying the same symbolic formula to point C, we have

$$v_C = \sqrt{-2g\Delta_{hAC}} = \sqrt{-2 \cdot 9.8\text{ m/s}^2 \cdot (2.00 - 5.00)\text{ m}} = 7.67\text{ m/s} \tag{8}$$

(b) The net work done by gravity is simply the change in gravitational potential energy with the sign reversed. So

$$W_g = -(U_{gC} - U_{gA}) = U_{gA} - U_{gC} = K_C \tag{9}$$
$$= -mg\Delta_{hAC} = -5.00\text{ kg} \cdot 9.8\text{ m/s}^2 \cdot (2.00 - 5.00)\text{ m} = 147\text{ J} \tag{10}$$

Problem 16. An object of mass m starts from rest and slides a distance d down a frictionless incline of angle θ. While sliding, it contacts an unstressed spring of negligable mass as shown in Figure P7.16. The object slides an additional distance x as it is brought momentarily to rest by the compression of the spring (of force constant k). Find the initial seperation d between the object and the spring.

This is just the symbolic form of Chapter 6 Problem 27 from last week's recitation.

There are no non-conservative forces, so

$$E_i = U_{gi} = mgh = E_f = U_{sf} = \frac{1}{2}kx^2 \tag{11}$$
$$h = (x+d) \cdot \sin\theta = \frac{kx^2}{mg} \tag{12}$$
$$d = \frac{kx^2}{mg\sin\theta} - x \tag{13}$$

Problem 22. In a needle biopsy, a narrow strip of tissue is extracted from a patient using a hollow needle. Rather than being pushed by hand, to ensure a clean cut the needle can be fired into the patient's body by a spring. Assume that the needle has mass $m = 5.60$ g, the light spring has force constant $k = 375$ N/m, and the spring is originally compressed $d_0 = 8.10$ cm to project the needle horizontally without friction. After the needle leaves the spring, the tip of the needle moves through $d_1 = 2.40$ cm of sking and soft tissue, which exerts a force $F_1 = 7.60$ N. Next, the needle cuts $d_2 = 3.50$ cm into an organ, which exerts on it a backward force of $F_2 = 9.20$ N. Find (a) the maximum speed of the needle and (b) the speed at which a flange on the back end of the needle runs into a stop that is set to limit the penetration to $p = 5.90$ cm.

Let us label the various points as follows.

Needle at rest, spring maximally compressed	A
Spring extended, needle just about to enter body	B
Needle at boundary between soft tissue and organ	C
Needle at maximal penetration into organ	D

(a) The needle will have its maximum speed at point B. The kinetic energy K_B at B will be equal to the spring potential energy U_{sA} at point A, because the launcher is frictionless, and there are no other relavent potentials. Therefore,

$$K_B = \frac{1}{2}mv_B^2 = U_{sA} = \frac{1}{2}kd_0^2 \tag{14}$$

$$v_B = d_0\sqrt{\frac{k}{m}} = 0.0810\text{ m}\sqrt{\frac{375\text{ N/m}}{5.60\cdot 10^{-3}\text{ kg}}} = 21.0\text{ m/s} \tag{15}$$

(b) The kinetic energy remaining in the needle just before the flange strikes the stop is given by

$$K_D = U_{sA} + W_1 + W_2 \tag{16}$$

Where W_1 and W_2 are the work done by the soft tissue and organ resistance respectively. In each case $W = -F\Delta_x$ because the forces are constant in the opposite direction to the motion of the needle. So

$$\frac{1}{2}mv_D^2 = \frac{1}{2}kd_0^2 - F_1 d_1 - F_2 d_2 \tag{17}$$

$$v_D = \sqrt{\frac{kd_0^2 - 2(F_1 d_1 + F_2 d_2)}{m}} \tag{18}$$

$$= \sqrt{\frac{375\text{ N/m}\cdot(0.0810\text{ m})^2 - 2\cdot(7.60\text{ N}\cdot 0.0240\text{ m} + 9.20\text{ N}\cdot 0.0350\text{ m})}{5.60\cdot 10^{-3}\text{ kg}}} \tag{19}$$

$$= 16.1\text{ m/s} \tag{20}$$

Problem 28. An $m_1 = 50.0$ kg block and an $m_2 = 100$ kg block connected by a string as shown in Figure P7.28. The pulley is frictionless and of negligible mass. The coefficient of kinetic friction between m_1 and the incline is $\mu = 0.250$. The incline is at an angle of $\theta = 37.0°$ from the horizontal. Determine the change in the kinetic energy of m_1 as it moces from point A to point B, a distance of $d = 20.0$ m.

Again, we use conservation of energy. Defining our gravitational potential energy to be zero at A we have

$$E_A + W_f = K_{1A} + K_{2A} + W_f = E_B = K_{1B} + K_{2B} + U_{g1B} + U_{g2B} \tag{21}$$

The blocks are tied together, so they must have the same velocity (since the string remains taught). So the change in velocity v is given by

$$\frac{1}{2}(m_1+m_2)v_A^2 - F_f d = \frac{1}{2}(m_1+m_2)v_B^2 + m_1 g d\sin\theta - m_2 g d \tag{22}$$

$$\Delta(v^2) = v_B^2 - v_A^2 = \frac{2}{m_1+m_2}\cdot[gd\cdot(m_2 - m_1\sin\theta) - F_f d] \tag{23}$$

$$\tag{24}$$

So the change in kinetic enery of m_1 is given by

$$\Delta(K_1) = \frac{dm_1}{m_1+m_2}\cdot[g(m_2 - m_1\sin\theta) - F_f] \tag{25}$$

We still need to find the force of fiction, which we do by constructing a free body diagram of m_1. We see that the forces on m_1 are friction $\mathbf{F}_f$, tension $\mathbf{T}$, normal $\mathbf{F}_N$, and gravitational $\mathbf{F}_{g1}$. Summing the forces in the direction perpendicular to the incline ($\mathbf{y}$), we have

$$\sum F_y = F_N - F_{g1}\cos\theta = 0 \tag{26}$$

$$F_N = m_1 g\cos\theta \tag{27}$$

The block is always sliding so $F_f = \mu F_N = \mu m_1 g \cos\theta$. Plugging this into our equation for $\Delta(K_1)$ we have

$$\Delta(K_1) = \frac{dgm_1}{m_1+m_2}\cdot(m_2 - m_1\sin\theta - \mu m_1\cos\theta) \tag{28}$$
$$= \frac{20.0\text{ m}\cdot 9.8\text{ m/s}^2\cdot 50.0\text{ kg}}{150\text{ kg}}\cdot[100\text{ kg} - 50\text{ kg}(\sin 27.0^\circ - 0.250\cos 27.0^\circ)] \tag{29}$$
$$= 5.78\text{ kJ} \tag{30}$$

Problem 54. An $m = 1.00$ kg object slides to the right on a surface having a coefficient of kinetic friction of $\mu = 0.250$ (Fig. P7.54). The object has a speed of $v_i = 3.00$ m/s when t makes contact with a light spring that has a force constant of $k = 50.0$ N/m (point A). The object comes to rest after the spring has been compressed a distance d (point B). The object is then forced toward the left by the spring and continues to move in that direction beyond the spring's unstretched position. The object finally comes to rest a distance D to the left of the unstretched spring (point D). Find (a) the distance of compression d, (b) the speed v at the unstretched position when the object is moving to the left (point C), and (c) the distance D where the object comes to rest.

(a) Conserving energy between points A and B

$$E_A + W_{AB} = \frac{1}{2}mv_i^2 - \mu mgd = E_B = \frac{1}{2}kd^2 \tag{31}$$
$$0 = \frac{k}{m}d^2 + 2\mu gd - v_i^2 \tag{32}$$
$$d = \frac{-2\mu g \pm\sqrt{(2\mu g)^2 - 4(k/m)(-v_i^2)}}{2k/m} \tag{33}$$
$$= \frac{-2\cdot 0.250\cdot 9.8\text{ m/s}^2 \pm\sqrt{(2\cdot 0.250\cdot 9.8\text{ m/s}^2)^2 + 4(50.0\text{ N/m}/1.00\text{ kg})(3.00\text{ m/s})^2}}{2\cdot 50.0\text{ N/m}/1.00\text{ kg}} \tag{34}$$
$$= (-0.0490 \pm 0.427)\text{ m} = 0.378\text{ m} \tag{35}$$

(b) Conserving energy between B and C

$$E_B + W_{BC} = \frac{1}{2}kd^2 - \mu mgd = E_B = \frac{1}{2}mv^2 \tag{36}$$
$$v = \sqrt{\frac{k}{m}d^2 - 2\mu gd} = \sqrt{\frac{50.0\text{ N/m}}{1.00\text{ kg}}(0.378\ldots\text{ m})^2 - 2\cdot 0.250\cdot 9.8\text{ m/s}^2\cdot 0.378\ldots\text{ m}} = 2.67\text{ m/s} \tag{37}$$

(c) Conserving energy between C and D

$$E_C + W_{CD} = \frac{1}{2}mv^2 - \mu mgD = E_D = 0\text{ J} \tag{38}$$
$$D = \frac{v^2}{2\mu g} = \frac{(2.67\ldots\text{ m/s})^2}{2\cdot 0.250\cdot 9.8\text{ m/s}^2} = 1.46\text{ m} \tag{39}$$

Problem 55. A block of mass $m = 0.500$ kg is pushed against a horizontal spring of negligable mass until the spring is compressed a distance x (Fig. P7.55) (point A). The force constant of the spring is $k = 450$ N/m. When it is released, the block travels along a frictionless, horizontal surface to point B, the bottom of a vertical circular track of radius $R = 1.00$ m, and continues to move up the track. The speed of the block at the bottom of the track is $v_B = 12.0$ m/s, and the block experiences an average friction force of $F_f = 7.00$ N while sliding up the track. (a) What is x? (b) What speed v_T do you predict for the block at the top of the track (point T)? (c) Does the block actually reach the top of the track, or does it fall off before reaching the top?

(a) Conserving energy between A and B

$$E_A = \frac{1}{2}kx^2 = E_B = \frac{1}{2}mv^2 \tag{40}$$
$$x = v\sqrt{\frac{m}{k}} = 12.0\text{ m/s}\sqrt{\frac{0.500\text{ kg}}{450\text{ N/m}}} = 0.400\text{ m} \tag{41}$$

(b) Conserving energy between B and T

$$E_B + W_f = \frac{1}{2}mv_B^2 - \pi RF_f = E_T = \frac{1}{2}mv_T^2 + mg2R \tag{42}$$
$$v_T = \sqrt{v_B^2 - 2R(\pi F_f/m + 2g)} = \sqrt{(12.0\text{ m/s})^2 - 2\cdot 1.00\text{ m}(\pi 7.00\text{ N}/0.500\text{ kg} + 2\cdot 9.8\text{ m/s}^2)} = 4.10\text{ m/s} \tag{43}$$

(c) The centerward acceleration of the block if it passes through T is

$$a_c = \frac{v_T^2}{r} \sim 16\ \text{m/s}^2 > g \tag{44}$$

So the block reaches the top and is still attached to the ramp, because it is still pushing out with some normal force against the track.

Problem 61. A pendulum, comprising a light string of length L and a small sphere, swings in a vertical plane. The string hits a peg located a distance d below the point of suspension (Fig. P7.61). (a) Show that if the sphere is released from a height below that of the peg (point A), it will return to this height after the string strikes the peg (point B). (b) Show that if the pendulum is released from the horizontal position ($\theta = 90°$) and is to swing in a complete circle centered on the peg, the minimum value of d must be $3L/5$.

(a) Conserving energy between points A and B (the sphere is at rest at both points).

$$E_A = mgh_A = E_B = mgh_B \tag{45}$$

$$h_A = h_B \tag{46}$$

(b) The radius of the smaller circle is $r = L - d$. The critical point is when sphere is in the vertical position of it's circle around the peg. The higher the peg is, the slower it's velocity will be at this point and the less the tension will be. At the smallest possible d for a complete circle, there will be no tension at this point and we can find the velocity with

$$a_c = g = \frac{v^2}{r} \tag{47}$$

$$v^2 = g(L-d) \tag{48}$$

Then conserving energy between this point D and the release point C (letting $h = 0$ be the level of the peg)

$$E_C = mgd = E_D = \frac{1}{2}mv^2 + mg(L-d) = \frac{1}{2}mg(L-d) + mg(L-d) \tag{49}$$

$$d = \frac{3}{2}(L-d) \tag{50}$$

$$\frac{5}{2}d = \frac{3}{2}L \tag{51}$$

$$d = \frac{3L}{5} \tag{52}$$