Recitation 6 solutions

Problem 30. *An $m = 2.00$ kg block is attached to a spring of force constant $k = 500$ N/m as shown in Active Figure 6.8 on page 164. The block is pulled $A = 5.00$ cm to the right of equilibrium and released from rest. Find the speed the block has as it passes through equilibrium if (a) the horizontal surface is frictionless and (b) the coefficient of friction between block and surface is $\mu = 0.350$.*

For both cases we will use conservation of energy. Call the point where the block is released P_0 and the point where the block passes through equilibrium P_1. At P_0, the block has spring potential energy $U_{s0} = 1/2 \cdot kA^2$ and no kinetic or gravitational potential energy. At P_1, the block has kinetic energy $K_1 = 1/2 \cdot mv^2$ and no potential energy.

(a) Without friction, the energy at P_1 is the same as that at P_0 because there is no energy lost to friction. So

$$P_0 = P_1 \frac{1}{2}kA^2 = \frac{1}{2}mv^2 \tag{1}$$

$$v = A\sqrt{\frac{k}{m}} = 5 \text{ cm}\sqrt{\frac{500 \text{ kg/s}^2}{2 \text{ kg}}} = 79.1 \text{ cm/s} \tag{2}$$

(b) With friction, part of the initial energy P_0 bleeds out into internal heat energy. The work done by friction is given by

$$W_f = \mathbf{F} \cdot \boldsymbol{\Delta}\mathbf{x} \tag{3}$$

Because the block is sliding the whole way in, the frictional force is always maxed out at the constant

$$F_f = \mu F_N = \mu mg \tag{4}$$

In the direction opposite to the motion. So friction from the table does

$$W_f = -F_f A = -\mu mgA \tag{5}$$

Where the negative sign denotes the frictional force sucking energy from the block.

Knowing the frictional work, the velocity at the equilibrium position is given by

$$E_0 + W_f = U_{s0} + W_f = E_1 = K_1 \tag{6}$$

$$\frac{1}{2}kA^2 - \mu mgA = \frac{1}{2}mv^2 \tag{7}$$

$$mv^2 = kA^2 - 2\mu mgA \tag{8}$$

$$v = \sqrt{\frac{k}{m}A^2 - 2\mu gA} \tag{9}$$

$$= \sqrt{\frac{500 \text{ kg/s}^2}{2 \text{ kg}}(0.05 \text{ m})^2 - 2 \cdot 0.35 \cdot 9.8 \text{ m/s}^2 \cdot 0.05 \text{ m}} \tag{10}$$

$$= 0.531 \text{ m/s} \tag{11}$$

What I was doing for (b) in class on Wednesday was more complicated because I had misread the question. I thought it was asking us to find the *maximum* speed, when it just asks for the speed at equilibrium. Figuring out when the maximum speed occurs requires more knowledge of differential equations than you guys are responsible for.

Problem 57. *In diatomic molecules, the consituent atoms exert attractive forces on each other at large distances, and repulsive forces at short distances. For many molecules, the Lennard-Jones law is a good approximation to the magnitude of these forces:*

$$F = F_0\left[2\left(\frac{\sigma}{r}\right)^{13} - \left(\frac{\sigma}{r}\right)^7\right] \tag{12}$$

Where r is the center-to-center distance between the atoms in the molecule, σ is a length parameter, and F_0 is the force when $r = \sigma$. For an oxygen molecule, $F_0 = 9.60 \cdot 10^{-11}$ N and $\sigma = 3.50 \cdot 10^{-10}$ m. Determine the work done by this force as the atoms are pulled apart from $r_0 = 4.00 \cdot 10^{-10}$ m to $r_1 = 9.00 \cdot 10^{-10}$ m.

The work done by the force is given by

$$
\begin{aligned}
W &= \int_{r_0}^{r_1} \mathbf{F} \cdot d\mathbf{r} && (13)\\
&= \int_{r_0}^{r_1} F \cdot dr && (14)\\
&= \int_{r_0}^{r_1} \left\{ F_0 \left[2\left(\frac{\sigma}{r}\right)^{13} - \left(\frac{\sigma}{r}\right)^{7} \right] \right\} \cdot dr && (15)\\
&= 2F_0 \int_{r_0}^{r_1} \left(\frac{\sigma}{r}\right)^{13} dr - F_0 \int_{r_0}^{r_1} \left(\frac{\sigma}{r}\right)^{7} dr && (16)\\
& && (17)
\end{aligned}
$$

Then we note that

$$
\begin{aligned}
\int \left(\frac{a}{x}\right)^n dx = a^n \int x^{-n} dx = a^n \frac{x^{-n+1}}{-n+1} && (18)\\
\int_{r_0}^{r_1} \left(\frac{a}{x}\right)^n dx = \frac{a^n}{1-n}\left(r_1^{1-n} - r_0^{1-n}\right) && (19)
\end{aligned}
$$

And plug this into our equation for W

$$
\begin{aligned}
W &= 2F_0 \frac{\sigma^{13}}{-12}\left(r_1^{-12} - r_0^{-12}\right) - F_0 \frac{\sigma^7}{-6}\left(r_1^{-6} - r_0^{-6}\right) && (20)\\
&= \frac{-F_0\sigma}{6}\left[\left(\frac{\sigma}{r_1}\right)^{12} - \left(\frac{\sigma}{r_1}\right)^{6}\right] + \frac{F_0\sigma}{6}\left[\left(\frac{\sigma}{r_0}\right)^{12} - \left(\frac{\sigma}{r_0}\right)^{6}\right] && (21)\\
&= \frac{-F_0\sigma}{6}\left[\sigma^{12}\left(r_1^{-12} - r_0^{-12}\right) - \sigma^6\left(r_1^{-6} - r_0^{-6}\right)\right] && (22)\\
&= \frac{-9.50 \cdot 10^{-11}\ \mathrm{N} \cdot 3.50 \cdot 10^{-10}\ \mathrm{m}}{6}\left\{(3.50\ \text{Å})^{12}\left[(9.00\ \text{Å})^{-12} - (4.00\ \text{Å})^{-12}\right] - (3.50\ \text{Å})^{6}\left[(9.00\ \text{Å})^{-6} - (4.00\ \text{Å})^{-6}\right]\right\} && (23)\\
&= 1.35 \cdot 10^{-25}\ \mathrm{J} && (24)
\end{aligned}
$$