Recitation 5 solutions

Problem 16. *In the Bohr model of the hydrogen atom, the speed of the electron is approximately $v = 2.20 \cdot 10^6$ m/s. Find (a) the force acting on the electron as it revolves in a circular orbit of radius $r = 0.530 \cdot 10^{-10}$ m and (b) the centripetal acceleration of the electron.*

Doing (b) first,

$$a_c = v^2/r = \frac{(2.20 \cdot 10^6 \text{ m/s})^2}{0.530 \cdot 10^{-10} \text{ m}} = 9.13 \cdot 10^{22} \text{ m/s}^2 \tag{1}$$

And going back to (a), (where the mass of an electron $m_e = 9.109 \cdot 10^{-31}$ kg came from the inside front cover of the text.)

$$F_c = m_e a_c = 9.109 \cdot 10^{-31} \text{ kg} \cdot 9.13 \cdot 10^{22} \text{ m/s}^2 = 8.32 \cdot 10^{-8} \text{ N} \tag{2}$$

Problem 24. *A roller coaster has vertical loops shaped like tear drops (Fig. P5.24). The cars ride on the inside of the loop at the top, and the speeds are high enough to ensure that the cars remain on the track. The biggest loop is $h = 40.0$ m high, with a maximum speed $v_b = 31.0$ m/s at the bottom. Suppose the speed at the top is $v_t = 13.0$ m/s and the corresponding centripetal acceleration is $a_{ct} = 2g$. (a) What is the radius r_t of the arc of the teardrop at the top? (b) If the total mass of a car plus the riders is M, what force F_N does the rail exert on the car at the top? (c) Suppose the roller coaster had a circular loop of radius $r = 20$ m. If the cars have the same speed v_t at the top, what is the centripetal acceleration a_{cc} at the top? Comment on the normal force at the top in this situation.*

(a)

$$a_{ct} = v_t^2/r_t \tag{3}$$

$$r_t = v_t^2/a_{ct} = (13.0 \text{ m/s})^2/(2 \cdot 9.8 \text{ m/s}^2) = 8.62 \text{ m} \tag{4}$$

(b) The central force $F_t = 2Mg$. This force is a combination of the force of gravity $F_g = Mg$ and the normal force F_N from the rail:

$$F_t = \sum F_{central} = F_g + F_N \tag{5}$$

$$F_N = F_t - F_g = 2Mg - Mg = Mg = 9.8 \text{ m/s}^2 \cdot M \tag{6}$$

(c)

$$a_{cc} = v_t^2/r = (13.0 \text{ m/s})^2/20 \text{ m} = 8.45 \text{ m/s}^2 \tag{7}$$

So the the new normal force F_{Nc} is going to be:

$$F_{Nc} = F_{cc} - F_g = (8.45 - 9.8) \text{ m/s}^2 \cdot M = -1.35 \text{ m/s}^2 \cdot M \tag{8}$$

Where the - sign indicates the normal force is the track pulling the car *away* from the center. The teardrop shape allows the loop to be 40 m high while always keeping the track's normal force in the center-ward direction.

Problem 32. *Find the order of magnitude of the gravitational force that you exert on another person $r = 2$ m away. In your solution, state the quantities you measure or estimate and their values.*

We'll be using Newton's law for gravitation (text p. 144):

$$F_g = G\frac{mM}{r^2} \tag{9}$$

with $G = 6.673 \cdot 10^{-11}$ Nm^2/kg^2.

We need to estimate m and M. Both bodies are people, so I'll use my weight for both:

$$m \approx M \approx 165 \text{ lbs} \cdot \left[\frac{1 \text{ kg}}{\sim 2 \text{ lbs}}\right] \approx 82.5 \text{ kg} \tag{10}$$

So

$$F_g \approx G\frac{m^2}{r^2} = G(m/r)^2 \approx 6.673 \cdot 10^{-11} \text{ Nm}^2/\text{kg}^2 \, (82.5 \text{ kg}/2 \text{ m})^2 = 1.14 \cdot 10^{-7} \text{ N} \approx 1 \cdot 10^{-7} \text{ N} \tag{11}$$

Where I reduced the answer to one sig. fig. because of my rough mass approximation.

Problem 34. *In a thundercloud, there may be electric charges of $q_t = +40.0$ C near the top of the cloud and $q_b = -40.0$ C near the bottom of the cloud. These charges are separated by $r = 2.00$ km. What is the electric force on the top charge?*

We'll be using Coulomb's law for the electro-magnetic force (text p. 144):

$$F_e = k_e \frac{q_1 q_2}{r^2} \tag{12}$$

with $k_e = 8.99 \cdot 10^9\ \mathrm{Nm^2/C^2}$.

So

$$F_e = 8.99 \cdot 10^9\ \mathrm{Nm^2/C^2} \frac{-40.0\ \mathrm{C} \cdot 40.0\ \mathrm{C}}{(2000\ \mathrm{m})^2} = -8.99 \cdot 10^9\ \mathrm{Nm^2/C^2} \left(\frac{40\ \mathrm{C}}{2000\ \mathrm{m}}\right)^2 = -3.596 \cdot 10^6\ \mathrm{N} \tag{13}$$

where the $-$ sign indicates an attractive force.

Problem 45. *A car rounds a banked curve as in Fig. 5.13. The radius of curvature of the road is R, the banking angle is /theta, and the coefficient of static friction is μ_s. (a) Determine the range of speeds the car can have without slipping up or down the road. (b) Find the minimum value of μ_s such that the minimum speed is zero. (c) What is the range of speeds possible if $R = 100$ m, $\theta = 10.0°$, and $\mu_s = 0.100$ (slippery conditions).*

Looking at Fig. 5.13 (text page 137) and adding friction, we see that the forces on the car are friction $\mathbf{F}_f$, gravity $\mathbf{F}_g$, and a normal force $\mathbf{F}_N$. Let the vertical direction be $\hat{\mathbf{j}}$ and the centerward direction to be $\hat{\mathbf{i}}$, and the direction centerward-down parallel to the surface of the road by $\hat{\mathbf{k}}$. Let us assume at first that $\mathbf{F}_N$ is in the $-\hat{\mathbf{k}}$ direction and at its maximum possible value of $F_f = \mu_s F_N$.

$$\begin{aligned}
\sum F_{\hat{\mathbf{j}}} &= F_N \cos\theta - mg + F_f \sin\theta = 0 && (14)\\
F_N(\cos\theta + \mu_s \sin\theta) &= mg && (15)\\
F_N &= \frac{mg}{\cos\theta + \mu_s\sin\theta} && (16)\\
\sum F_{\hat{\mathbf{i}}} &= F_N \sin\theta - F_f\cos\theta = F_N(\sin\theta - \mu_s\cos\theta) && (17)\\
&= \frac{mg}{\cos\theta + \mu_s\sin\theta}(\sin\theta - \mu_s\cos\theta) && (18)\\
&= mg\frac{\tan\theta - \mu_s}{1+\mu_s\tan\theta} = m\frac{v^2}{R} && (19)\\
v &= \sqrt{Rg\frac{\tan\theta - \mu_s}{1+\mu_s\tan\theta}} && (20)
\end{aligned}$$

(a) The work above shows that the minimum speed a car can have while going around the turn is given by eqn 20, because that is the case when friction is maximized in the $-\hat{\mathbf{k}}$ direction. The maximum speed that the car can have can be found by simply reversing the sign of the frictional force above (so that $\mathbf{F}_f$ points in the $+\hat{\mathbf{k}}$ direction), which we achieve by replacing any μ_ss in eqn 20 with $(-\mu_s)$. For any speeds between these F_f will be less than its maximum value of $\mu_s F_N$, and the car will still not slip. So

$$\sqrt{Rg\frac{\tan\theta - \mu_s}{1+\mu_s\tan\theta}} \le v \le \sqrt{Rg\frac{\tan\theta + \mu_s}{1-\mu_s\tan\theta}} \tag{21}$$

(b) If the speed is 0, then $\mathbf{F}_f$ will be in the $-\hat{\mathbf{k}}$ direction (opposing the $+\hat{\mathbf{k}}$ portion of $\mathbf{F}_g$). Summing the forces in the $\hat{\mathbf{k}}$direction we have

$$\begin{aligned}
\sum F_{\hat{\mathbf{k}}} &= F_g\sin\theta - F_N = mg(\sin\theta - \mu_s\cos\theta) = 0 && (22)\\
\mu_s &= \tan\theta && (23)
\end{aligned}$$

Or we could go use eqn 19, our sum of forces in the $\hat{\mathbf{i}}$ direction.

$$\sum F_{\hat{\mathbf{i}}} = mg\frac{\tan\theta - \mu_s}{1+\mu_s\tan\theta} = m\frac{v^2}{R} = 0 \tag{24}$$

And set the numerator to 0, which gives the same formula for μ_s.

(c) Plugging into our ans for (a) we have

$$\begin{aligned}
\sqrt{100\ \mathrm{m}\cdot 9.8\ \mathrm{m/s}^2\frac{\tan 10.0° - 0.100}{1+0.100\cdot\tan 10.0°}} \le v \le \sqrt{100\ \mathrm{m}\cdot 9.8\ \mathrm{m/s}^2\frac{\tan 10.0° + 0.100}{1-0.100\cdot\tan 10.0°}} && (25)\\
8.57\ \mathrm{m/s} \le v \le 16.6\ \mathrm{m/s} && (26)
\end{aligned}$$

Problem 47. *In a home laundry dryer, a cylindrical tub containing wet clothes is rotated steadily about a horizontal axis as shown in Fig. P5.47. The clothes are made to tumble so that they will dry uniformly. The rate of rotation of the smooth-walled tub is chosen so that a small piece of cloth will lose contact with the tub when the cloth is at an angle of the* $\theta = 68.0^o$ *above the horizontal. If the radius of the tub is* $r = 0.330$ *m, what rate of revolution is needed?*

Looking at the figure, we see that there are fins sticking out of the drum wall, so that clothes do not slip along the surface. Because of this, we can ignore forces in the tangential direction. Focusing on the center-ward direction, we see that the angle between the force of gravity $F_g = mg$ and the center-ward direction is

$$\theta' = 90.0^o - \theta = 90.0^o - 68.0^o = 22.0^o \tag{27}$$

The sum of forces in the center-ward direction is then

$$F_c = F_g \cos\theta' = mg\cos\theta' \tag{28}$$

In order for this center-ward force to provide separation from the drum, this force must be the center-ward force needed for uniform circular motion

$$F_c = mv^2/r = mg\cos\theta' \tag{29}$$

$$v^2/r = g\cos\theta' \tag{30}$$

$$v = \sqrt{rg\cos\theta'} \tag{31}$$

and the frequency of rotation f is given by

$$f = \frac{v}{2\pi r} = \frac{\sqrt{rg\cos\theta'}}{2\pi r} = \frac{1}{2\pi}\sqrt{\frac{g\cos\theta'}{r}} = \frac{1}{2\pi}\sqrt{\frac{9.8\ \mathrm{m/s}^2\cos 22.0^o}{0.330\ \mathrm{m}}} = 0.835\ \mathrm{Hz} \tag{32}$$