Homework 9

Chapter 10

Problem 13. *A car traveling on a flat (unbanked) circular track accelerates uniformly from rest with a tangential acceleration of* $a_t = 1.70$ *m/s*^{2}*. The car makes it one forth of the way around the circle before it skids off the track. Determine the coefficient of static friction between the car and track from these data.*

In order to find the coefficient of static friction, we must examine the force of friction at the slipping point where $F_f = \mu_s F_N = \mu_s mg$. We don't know the mass of the car, but hopefully it will cancel out somewhere along the way. The only force on the car that is not completely in the vertical direction is friction, so let us consider the sums of forces in the tangential and centerward directions. First the tangential direction

$$\sum F_t = F_{ft} = ma_t \tag{1}$$

And then in the centerward direction

$$\sum F_c = F_{fc} = ma_c = m\frac{v_t^2}{r} \tag{2}$$

Going back to our constant acceleration equations we see that

$$v_t^2 = v_{ti}^2 + 2a_t\Delta x = 2a_t\frac{\pi r}{2} \tag{3}$$

So going backwards and plugging in

$$F_{fc} = m\frac{2a_t\pi r}{2r} = \pi m a_t \tag{4}$$

$$F_f = \sqrt{F_{ft}^2 + F_{fc}^2} = ma_t\sqrt{1+\pi^2} \tag{5}$$

$$\mu_s = \frac{F_f}{mg} = \frac{a_t}{g}\sqrt{1+\pi^2} = \frac{1.70\text{ m/s}^2}{9.80\text{ m/s}^2}\sqrt{1+\pi^2} = 0.572 \tag{6}$$

Problem 21. *Find the net torque* τ *on the wheel in Fig. P10.21 about the axle through O, taking* $a = 10.0$ *cm and* $b = 25.0$ *cm.*

Torque is defined as $\tau = \mathbf{r}\times\mathbf{F}$, so we have (defining the counter clockwise direction to be positive)

$$\sum \tau = -b\cdot 10.0\text{ N} + a\cdot 12.0\text{ N} - b\cdot 9.00\text{ N} \tag{7}$$

Where the $-$ sign on the first and third terms denote torques in the $-$ direction. There are no sin terms, because all three forces are in the tangential direction. The 12.0 N force is slightly suspicious, since they tell you it makes an angle of 30° with the horizontal, but if you look closely, you'll see that it isn't actually applied to the top of the circle, and it *is* tangential to it's application radius.

Plugging in for a and b we have

$$\sum \tau = -0.250\text{ m}\cdot 10.0\text{ N} + 0.100\text{ m}\cdot 12.0\text{ N} - 0.250\text{ m}\cdot 9.00\text{ N} = -3.55\text{ J} \tag{8}$$

Problem 44. *A space station is constructed in the shape of a hollow ring of mass* $m = 5.00\cdot 10^4$ *kg. Members of the crew walk on a deck formed by the inner surface of the outer cylindrical wall of the ring, with a radius of* $r = 100$ *m. At rest when constructed, the ring is set rotating about its axis so that the people inside experience an effective free-fall acceleration equal to g. The rotation is achieved by firing two small rockets attached tangentially at opposite points on the outside of the ring. (a) What angular momentum does the space station acquire? (b) How long must the rockets be fired if each exerts a thrust of* $F = 125$ *N? (c) Prove that the total torque on the ring, multiplied by the the time interval found in (b), is equal to the change in angular momentum found in (a). This equality represents the* angular impulse-angular momentum theorem.

(a) The certerward acceleration of people on the wall of the space station is given by

$$g = a_c = \frac{v^2}{r} = r\omega^2 \tag{9}$$

$$\omega = \sqrt{\frac{g}{r}} \tag{10}$$

Where we used $v = r\omega$ to replace the linear velocity v. The moment of inertia of a ring is given by $I = mr^2$ from table 10.2 on page 300. The angular momentum is then given by

$$L = I\omega = mr^2\sqrt{\frac{g}{r}} = mr\sqrt{gr} = 1.57\cdot 10^8\text{ Js} \tag{11}$$

(b) The torque on the station is given by

$$\sum \tau = 2 \cdot r \cdot F = I\alpha = mr^2\alpha \tag{12}$$

$$\alpha = \frac{2F}{mr} \tag{13}$$

Going back to our constant acceleration equations, we see that

$$\omega = \alpha t + \omega_0 = \alpha t \tag{14}$$

$$t = \frac{\omega}{\alpha} = \sqrt{\frac{g}{r}} \cdot \frac{mr}{2F} = \sqrt{gr}\frac{m}{2F} = \sqrt{9.80 \text{ m/s}^2 \cdot 100 \text{ m}}\frac{5 \cdot 10^4 \text{ kg}}{2 \cdot 125 \text{ N}} = 6.26 \text{ ks} = 1.74 \text{ hr} \tag{15}$$

(c)

$$\tau t = I\alpha t = I\omega = L \tag{16}$$

$$2rFt = 2 \cdot 100 \text{ m} \cdot 125 \text{ N} \cdot 6.26 \cdot 10^3 \text{ s} = 1.57 \cdot 10^8 \text{ Js} = L \tag{17}$$

So they are equal both symbolically and numerically which means I probably didn't make any algebra mistakes (we can hope).

Problem 72. *A wad of sticky clay with mass m and velocity* $\mathbf{v}_i$ *is fired at a solid cylinder of mass M and radius R (Fig. P10.72). The cylinder is initially at rest and is mounted on a fixed horizontal axle that runs through its center of mass. The line of motion of the projectile is perpendixular to the axle and at a distance* $d < R$ *from the center. (a) Find the angular speed of the system just after the clay strikes and sticks to the surface of the cylinder. (b) Is the mechanical energy of the clay-cylinder system conserved in this process? Explain your answer.*

(a) Conserving angular momentum (letting the $\hat{\mathbf{i}}$ direction be into the page)

$$\mathbf{L}_i = \mathbf{R} \times \mathbf{p} = Rp\sin\theta\hat{\mathbf{i}} = Rmv_i\frac{d}{R}\hat{\mathbf{i}} = mv_i d\hat{\mathbf{i}} \tag{18}$$

$$= \mathbf{L}_f = I\omega\hat{\mathbf{i}} = \left(\frac{1}{2}MR^2 + mR^2\right)\omega\hat{\mathbf{i}} = \left(\frac{M}{2} + m\right)R^2\omega\hat{\mathbf{i}} \tag{19}$$

$$\omega = v_i\frac{md}{I_{tot}} \tag{20}$$

Where $I_{tot} \equiv (M/2 + m)R^2$.

(b) The change in energy is

$$\Delta K = K_f - K_i = \frac{1}{2}I_{tot}\omega^2 - \frac{1}{2}mv_i^2 \tag{21}$$

$$= \frac{1}{2}\left(\frac{m^2v_i^2d^2}{I_tot} - mv_i^2\right) = \frac{1}{2}mv_i^2\left(\frac{md^2}{I_tot} - 1\right) \tag{22}$$

$$= K_i\left(\frac{md^2}{(M/2+m)R^2} - 1\right) \tag{23}$$

$$\tag{24}$$

So for maximum final energy $d = R$ and the 2nd term on the right hand side reduces to

$$\frac{m}{M/2+m} - 1 = \frac{m - (M/2+m)}{M/2+m} = \frac{-M}{M+2m} < 0 \tag{25}$$

So the final energy is less than the initial energy unless $m = 0$, in which case the cylinder just sits still for eternity. The lost energy goes to the same types of internal energy that we had in Problem 24 in Chapter 8: warmer clay, thwacking sound, etc.