Homework 8

Problem 6. *A friend claims that as long as he has his seat belt on, he can hold on to an $m = 12.0$ kg child in a $v_i = 60.0$ mph head-on collision with a brick wall in which the car passenger compartment comes to a stop in $\Delta t = 0.050$ s. Show that the violent force during the collision will tear the child from his arms.*

The force needed to hold on to the child is given by

$$F = \frac{\Delta p}{\Delta t} = -m\frac{v_i}{\Delta t} = -12.0 \text{ kg}\frac{60 \text{ mph}}{0.050 \text{ s}} \cdot \frac{1609 \text{ m}}{1 \text{ mi}} \cdot \frac{1 \text{ hr}}{3600 \text{ s}} = -6436 \text{ N} \tag{1}$$

Which is much larger than what the friend is capable of applying.

Problem 24. *An $m_1 = 90$ kg fullback running east ($\hat{\mathbf{i}}$) with a speed of $v_1 = 5.00$ m/s is tackled by an $m_2 = 95$ kg opponent running north ($\hat{\mathbf{j}}$) with a speed of $v_2 = 3.00$ m/s. Noting that the collision is perfectly inelastic, (a) calculate the speed v_f and direction θ of the players just after the tackle and (b) determine the mechanical energy lost as a result of the collision. Account for the missing energy.*

(a) Conserving momentum in the $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ directions

$$P_{ix} = m_1 v_1 = P_{fx} = (m_1 + m_2)v_{fx} \tag{2}$$
$$v_{fx} = v_1 \frac{m_1}{m_1 + m_2} = 2.43 \text{ m/s} \tag{3}$$
$$P_{iy} = m_2 v_2 = P_{fy} = (m_1 + m_2)v_{fy} \tag{4}$$
$$v_{fy} = v_2 \frac{m_2}{m_1 + m_2} = 1.54 \text{ m/s} \tag{5}$$
$$v_f = \sqrt{v_{fx}^2 + v_{fy}^2} = 2.88 \text{ m/s} \tag{6}$$
$$\theta = \arctan\left(\frac{v_{fy}}{v_{fx}}\right) = 32.3^\circ \tag{7}$$

(b)

$$\Delta K = K_f - K_i = \left(\frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2\right) - \frac{1}{2}(m_1 + m_2)v_f^2 \tag{8}$$
$$= \frac{1}{2}\left[90.0 \text{ kg}(5.00 \text{ m.s})^2 + 95.0 \text{ kg}(3.00 \text{ kg})^2 - (90.0 \text{ kg} + 95.0 \text{ kg})(2.88 \text{ m/s})^2\right] \tag{9}$$
$$= -786 \text{ J} \tag{10}$$

All of which has been lost as mechanical energy, and is now thermal energy (warmer football players), noise (a loud crunch), etc.

Problem 28. *A proton, moving with a velocity of $v_i\hat{\mathbf{i}}$, collides elastically with another proton that is initially at rest. Assuming that the two protons have equal speeds after the collision, find (a) the speed v_f of each proton after the collision in terms of v_i and (b) the directions of the velocity vectors after the collision.*

(a) Looking at the front inside cover of the text we see that the the mass of a proton is given by $m_p = 1.672 \cdot 10^{-27}$ kg. Conserving energy (because the collision is elastic) we have

$$K_i = \frac{1}{2}m_p v_i^2 = K_f = \frac{1}{2}m_p v_f^2 + \frac{1}{2}m_p v_f^2 \tag{11}$$
$$v_f = \sqrt{\frac{v_i^2}{2}} = \frac{v_i}{\sqrt{2}} \tag{12}$$

(b) Let $\mathbf{v}_{f1}$ be the final velocity for the incident proton, and $\mathbf{v}_{f2}$ be the final velocity for the proton initially at rest. Conserving momentum in the $\hat{\mathbf{j}}$ direction

$$P_{iy} = 0 = P_{fy} = m_p v_{f1y} + m_p v_{f2y} \tag{13}$$
$$v_{f1y} = -v_{f2y} \tag{14}$$

So the protons have equal magnitude speeds in the $\hat{\mathbf{j}}$ direction. Because the speed of the particles are equal, the magnitude of their speeds in the $\hat{\mathbf{i}}$ direction should also be equal $|v_{f1x}| = |v_{f2x}|$. Conserving momentum in the $\hat{\mathbf{i}}$ direction.

$$P_{ix} = m_p v_i = P_{fx} = m_p v_{f1x} + m_p v_{f2x} = 2m_p v_{fx} \tag{15}$$
$$v_{fx} = \frac{v_i}{2} \tag{16}$$

Using the Pythagorean theorem to solve for the magnitude of v_{fy}

$$v_f^2 = \frac{v_i^2}{2} = v_{fx}^2 + v_{fy}^2 = \frac{v_i^2}{4} + v_{fy}^2 \tag{17}$$

$$v_{fy} = v_i\sqrt{\frac{1}{2} - \frac{1}{4}} = \frac{v_i}{2} = v_{fx} \tag{18}$$

So because the $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ components of $\mathbf{v}_f$ are the same, both protons are deflected away at an angle of $\theta = 45°$ from the $\hat{\mathbf{i}}$ direction, with opposite $\hat{\mathbf{j}}$ components (so the angle between $\mathbf{v}_{f1}$ and $\mathbf{v}_{f2}$ is $90°$).

Problem 51. *A small block of mass $m_1 = 0.500$ kg is released from rest at the top of a curve-shaped, frictionless wedge of mass $m_2 = 3.00$ kg, which sits on a frictionless, horizontal surface as sown in Fig. P8.51a. When the block leaves the wedge, its velocity is measured to be $v_1 = 4.00$ m/s to the right ($\hat{\mathbf{i}}$) as shown in Fig. P8.51b. (a) What is the velocity $\mathbf{v}_2$ of the wedge after the block reaches the horizontal surface? (b) What is the height h of the wedge?*

(a) Conserving momentum

$$P_i = 0 = P_f = m_1 v_1 + m_2 v_2 \tag{19}$$

$$v_2 = -v_1 \frac{m_1}{m_2} = -4.00 \text{ m/s} \frac{0.500 \text{ kg}}{3.00 \text{ kg}} = -0.667 \text{ m/s} \tag{20}$$

Where the $-$ sign denotes motion in the $-\hat{\mathbf{i}}$ direction (to the left in Fig. P8.51b).

(b) Let $y = 0$ be the level of the table for the purpose of calculating gravitational potential energy. Conserving energy (since none is lost to friction or other internal energies)

$$E_i = m_1 g h = E_f = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 \tag{21}$$

$$m_1 g h = \frac{1}{2}\left(m_1 v_1^2 + m_2 \frac{m_1^2 v_2^2}{m_2^2}\right) \tag{22}$$

$$= \frac{m_1 v_1^2}{2}\left(1 + \frac{m_1}{m_2}\right) \tag{23}$$

$$h = \frac{v_1^2}{2g}\left(1 + \frac{m_1}{m_2}\right) = \frac{(0.667 \text{ m/s})^2}{2 \cdot 9.80 \text{ m/s}^2}\left(1 + \frac{0.500 \text{ kg}}{3.00 \text{ kg}}\right) = 0.952 \text{ m} \tag{24}$$