Homework 7

Problem 4. *At 11:00AM on September 7, 2001, more than one million British school children jumped up and down for one minute. The curriculum focus of the "Giant Jump" was on earthquakes, but it was integrated with many other topics, such as exercise, geography, cooperation, testing hypothesis, ans setting world records. Children built their own seismographs that registered local effects. (a) Find the mechanical energy released in the experiment. Assume that $N_c = 1{,}050{,}000$ children of an average mass $m = 36.0$ kg jump $N_j = 12$ times each, raising their centers of mass by $h = 25.0$ cm each time and briefly resting between one jump and the next. The free gall acceleration in Britain is $g = 9.81$ m/s^2. (b) Most of the energy is converted very rapidly into internal energy within the bodies of the children and the floors of the school buildings. Of the energy that propagates into the ground, most produces high frequency "microtremor" vibrations that are rapidly damped and cannot travel far. Assume that $p = 0.01$ % of the energy is carried away by a long-range seismic wave. The magnitude of an earthquake on the Richter scale is given by*

$$M = \frac{\log E - 4.8}{1.5} \tag{1}$$

Where E is the seismic wave energy in joules. According to this model, what is the magnitude of the demonstration quake? It did not register above background noise overseas or on the seismograph of the Wolverton Seismic Vault, Hampshire.

(a) From "briefly resting between each jump" we are to conclude that each collision is perfectly inelastic (that all the mechanical energy the student had during the jump was lost into internal energies). The energy lost by one student preforming a single jump is just $U_g = mgh$, so the energy lost during the entire experiment is

$$E_T = N_c N_j mgh = 1.05 \cdot 10^6 \cdot 12 \cdot 36.0 \text{ kg} \cdot 9.81 \text{ m/s}^2 \cdot 0.250 \text{ m} = 1.11 \cdot 10^9 \text{ J} \tag{2}$$

(b) The energy in long-range seismic waves is given by $E = pE_t/100$ so the magnitude of the "quake" is

$$M = \frac{\log(0.01 \cdot 1.11 \cdot 10^9) - 4.8}{1.5} = 0.164 \tag{3}$$

Problem 47. *The system shown in Fig. P7.47 consists of a light inextensible cord, light frictionless pulleys, and blocks of equal mass. It is initially held at rest so that the blocks are at the same height above the ground. The blocks are then released. Find the speed of block A at the moment when the vertical separation of the blocks is h.*

Let m be the mass of one block, and $\hat{\mathbf{i}}$be the vertical direction, with $x = 0$ for both blocks at their initial position. After some consideration, we decide that block A will fall and block B will rise (if you are not convinced, find the mass that B must have in order for the system to remain stationary). In order to get a quantitative relationship between the motion of the two blocks, imagine that B moves up a distance x in some time Δt. Then block B will have an average velocity of $v_B = x/\Delta t$, and block A will have gone a distance $-2x$ with an average velocity of $v_A = -2x/\Delta t$. So $x_A = -2x_B$ and $v_A = -2v_B$.

When they are a distance h apart

$$h = x_B - x_A = x_B + 2x_B = 3x_B \tag{4}$$

$$x_B = \frac{h}{3} \tag{5}$$

$$x_A = \frac{-2h}{3} \tag{6}$$

So conserving energy

$$E_i = K_i + U_{gi} = 0 = E_f = K_f + U_{gf} = \frac{1}{2}mv_A^2 + \frac{1}{2}mv_B^2 + mg\frac{h}{3} + mg\frac{-2h}{3} \tag{7}$$

$$g\frac{2h}{3} = v_A^2 + v_B^2 = v_A^2 + \left(\frac{-v_A}{2}\right)^2 = v_A^2\left(1 + \frac{1}{4}\right) = \frac{5}{4}v_A^2 \tag{8}$$

$$v_A = \sqrt{\frac{8gh}{15}} \tag{9}$$

Problem 50. *A child's pogo stick (Fig/ P7.50) stores energy in a spring with a force constant of $k = 2.50 \cdot 10^4$ N/m. At position A ($x_A = -0.100$ m), the spring compression is a maximum and the child is momentarily at rest. At position B ($x_B = 0$), the spring is relaxed and the child is moving upward. At position C, the child is again momentarily at rest at the top of the jump. The combined mass of child and pogo stick is $m = 25.0$ kg. (a) Calculate the total energy of the child-stick-Earth system, taking both gravitational and elastic potential energy as zero for $x = 0$. (b) Determine x_C. (c) Calculate the speed of the child at B. (d) Determine the value of x for which the kinetic energy of the system is a maximum. (e) Calculate the child's maximum upward speed.*

(a) We know the most about point A, so we'll calculate the total energy there.

$$E_A = U_{sA} + U_{gA} = \frac{1}{2}kx_A^2 + mgx_A = \frac{1}{2}\cdot 2.50\cdot 10^4\text{ N/m}\cdot(-0.100\text{ m})^2 + 25.0\text{ kg}\cdot 9.8\text{ m/s}^2\cdot(-0.100\text{ m}) = 100.5\text{ J} \tag{10}$$

(b) Conserving energy between A and C

$$E_A = E_C = U_{gC} = mgx_C \tag{11}$$

$$x_C = \frac{E_A}{mg} = \frac{100.5\text{ J}}{25.0\text{ kg}\cdot 9.8\text{ m/s}^2} = 0.410\text{ m} \tag{12}$$

(c) Conserving energy between A and B

$$E_A = E_B = K_B = \frac{1}{2}mv_B^2 \tag{13}$$

$$v_B = \sqrt{\frac{2E_A}{m}} = \sqrt{\frac{2\cdot 100.5\text{ J}}{25.0\text{ kg}}} = 2.84\text{ m/s} \tag{14}$$

(d) The kinetic energy is maximized when the speed is maximized which occurs at the point where the accelerating spring force balances the decelerating gravitational force. Before this point, the spring force exceeded the gravitational force and the child was speeding up. Afterward, the gravitation force exceeded the spring force and the child was slowing down.

$$F_s = -kx = -F_g = mg \tag{15}$$

$$x = \frac{mg}{k} = \frac{25.0\text{ kg}\cdot 9.8\text{ m/s}^2}{2.50\cdot 10^4\text{ N/m}} = -0.00980\text{ m} = -9.80\text{ mm} \tag{16}$$

(e) Conserving energy between A and the point of maximum velocity D

$$E_A = E_D = U_{sD} + U_{gD} + K_D \tag{17}$$

$$K_D = \frac{1}{2}mv_D^2 = E_A - U_{sD} - U_{gD} = E_A - \frac{1}{2}kx_D^2 - mgx_D \tag{18}$$

$$v_D = \sqrt{\frac{2E_A - kx_D^2 - 2mgx_D}{m}} \tag{19}$$

$$= \sqrt{\frac{2\cdot 100.5\text{ J} - 2.50\cdot 10^4\text{ N/m}\cdot(-9.80\cdot 10^{-3}\text{ m})^2 - 2\cdot 25.0\text{ kg}\cdot 9.8\text{ m/s}^2\cdot(-9.80\cdot 10^{-3}\text{ m})}{25.0\text{ kg}}} \tag{20}$$

$$= 2.85\text{ m/s} \tag{21}$$

Problem 62. *A roller coaster car is released from rest at the top of the first rise and then moves freely with negligible friction. The roller coaster shown in Fig. P7.62 has a circular loop of radius R in the vertical plane. (a) First, suppose the car barely makes it around the loop; at the top of the loop the riders are upside down and feel weightless. Find the required height of the release point above the bottom of the loop, in terms of R. (b) Now assume that the release point is at or above the minimum required height. Show that the normal force on the car at the bottom of the loop exceeds the normal force at the top by six times the weight of the car. The normal force on each rider follows the same rule. Such a large normal force is dangerous and very uncomfortable for the riders. Roller coasters are therefore not build with circular loops in vertical planes. Figure P5.24 and the photograph on page 134 show two actual designs.*

(a) Because the riders "feel weightless" at the top of the loop (point T), we will assume that they are in free fall with a centerwards acceleration of $g = v_T^2/R$. Conserving energy between T and the release point A

$$E_A = mgh = E_T = \frac{1}{2}mv_T^2 + mg(2R) \tag{22}$$

$$h = \frac{1}{2g}v_T^2 + 2R = \frac{Rg}{2g} + 2R = 2.5R \tag{23}$$

(b) If the release comes from a higher point, there will be some normal force at the top N_T and at the bottom N_B. Summing forces at both points

$$\sum F_{cT} = mg + N_T = m\frac{v_T^2}{R} \tag{24}$$

$$v_T^2 = gR + R\frac{N_T}{m} \tag{25}$$

$$\sum F_{cB} = -mg + N_B = m\frac{v_B^2}{R} \tag{26}$$

$$v_B^2 = -gR + R\frac{N_B}{m} \tag{27}$$

And conserving energy between the top and bottom

$$E_B = \frac{1}{2}mv_B^2 = E_T = \frac{1}{2}mv_T^2 + mg(2R) \tag{28}$$

$$v_B^2 = v_T^2 + 4gR = -gR + R\frac{N_B}{m} = gR + R\frac{N_T}{m} + 4gR \tag{29}$$

$$N_B = N_T + 6mg \tag{30}$$