Homework 6

Problem 9. *Using the definition of the scalar product, find the angles between (a)* $\mathbf{A} = 3\hat{\mathbf{i}} - 2\hat{\mathbf{j}}$ *and* $\mathbf{B} = 4\hat{\mathbf{i}} - 4\hat{\mathbf{j}}$, *(b)* $\mathbf{A} = -2\hat{\mathbf{i}} + 4\hat{\mathbf{j}}$ *and* $\mathbf{B} = 3\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$, *and (c)* $\mathbf{A} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$ *and* $\mathbf{B} = 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$.

From the definition of the scalar (or dot) product on pages 160 and 161, we see

$$\mathbf{A} \cdot \mathbf{B} = AB\cos\theta = A_x B_x + A_y B_y + A_z B_z \tag{1}$$
$$\theta = \arccos\left(\frac{A_x B_x + A_y B_y + A_z B_z}{AB}\right) \tag{2}$$

(a)

$$A = \sqrt{3^2 + 2^2} = \sqrt{9 + 4} = \sqrt{13} \tag{3}$$
$$B = \sqrt{4^2 + 4^2} = 4\sqrt{2} \tag{4}$$
$$\sum A_i B_i = 3 \cdot 4 + (-2) \cdot (-4) = 12 + 8 = 20 \tag{5}$$
$$\theta = \arccos\left(\frac{20}{4\sqrt{26}}\right) = 11.3^\circ \tag{6}$$

(b)

$$A = \sqrt{2^2 + 4^2} = \sqrt{4 + 16} = \sqrt{20} \tag{7}$$
$$B = \sqrt{3^2 + 4^2 + 2^2} = \sqrt{9 + 16 + 4} = \sqrt{29} \tag{8}$$
$$\sum A_i B_i = (-2) \cdot 3 + 4 \cdot (-4) = -6 + -16 = -22 \tag{9}$$
$$\theta = \arccos\left(\frac{-22}{\sqrt{580}}\right) = 156^\circ \tag{10}$$

(c)

$$A = \sqrt{1^2 + 2^2 + 2^2} = \sqrt{1 + 4 + 4} = \sqrt{9} = 3 \tag{11}$$
$$B = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5 \tag{12}$$
$$\sum A_i B_i = (-2) \cdot 3 + 2 \cdot 4 = -6 + 8 = 2 \tag{13}$$
$$\theta = \arccos\left(\frac{2}{15}\right) = 82.3^\circ \tag{14}$$

Problem 24. *An* $m = 4.00$ *kg particle is subject to a total force that varies with position as shown in Fig. P6.11. The particle starts from rest at* $x = 0$. *What is the speed at (a)* $x = 5.00$ *m, (b)* $x = 10.0$ *m, and (c)* $x = 15.0$ *m?*

In each of these cases we'll be conserving energy. The energy put in by the force all goes into the particle's kinetic energy. So conserving energy we have

$$W_{0,5} = \int_{0\text{ m}}^{5\text{ m}} \mathbf{F} \cdot \mathbf{dx} = \frac{1}{2} m v^2 \tag{15}$$
$$v_5 = \sqrt{\frac{2W_{0,5}}{m}} = \sqrt{\frac{2 \cdot 1.5\text{ N} \cdot 5\text{ m}}{4.00\text{ kg}}} = 1.94\text{ m/s} \tag{16}$$

Doing the same for the other works

$$W_{0,10} = W_{0,5} + 3\text{ N} \cdot 5\text{ m} = 22.5\text{ J} \tag{17}$$
$$W_{0,15} = W_{0,10} + 1.5\text{ N} \cdot 5\text{ m} = 30\text{ J} \tag{18}$$

So the other velocities are

$$v_{10} = \sqrt{\frac{2W_{0,10}}{m}} = 3.35\text{ m/s} \tag{19}$$
$$v_{15} = \sqrt{\frac{2W_{0,15}}{m}} = 3.87\text{ m/s} \tag{20}$$

Problem 29. *An $m = 40.0$ kg box initially at rest is pushed $x = 5.00$ m along a rough, horizontal floor with a constant applied horizontal force of $F = 130$ N. The coefficient of friction between box and floor is $\mu = 0.300$. Find (a) the work done by the applied force, (b) the increase in internal energy in the box-floor sysem as a result of the friction, (c) the work done by the normal force, (d) the work done by the gravitational force, (e) the change in kinetic energy of the box, and (f) the final speed of the box.*

(a)

$$W_F = \mathbf{F} \cdot \mathbf{x} = 130 \text{ N} \cdot 5.00 \text{ m} = 650 \text{ J} \tag{21}$$

(b)

$$U_f = -\mathbf{F}_f \cdot \mathbf{x} = \mu mgx = 0.300 \cdot 40.0 \text{ kg} \cdot 9.8 \text{ m/s}^2 \cdot 5.00 \text{ m} = 588 \text{ J} \tag{22}$$

(c)

$$W_N = \mathbf{F}_N \cdot \mathbf{x} = 0 \text{ J} \tag{23}$$

(d)

$$W_g = \mathbf{F}_g \cdot \mathbf{x} = 0 \text{ J} \tag{24}$$

(e) Conserving energy

$$W_F = K_f + U_f \tag{25}$$

$$\Delta K = W_F - U_f = (650 - 588) \text{ J} = 62 \text{ J} \tag{26}$$

(f)

$$\frac{1}{2} m v^2 = \Delta K \tag{27}$$

$$v = \sqrt{\frac{2\Delta K}{m}} = \sqrt{\frac{2 \cdot 62 \text{ J}}{40.0 \text{ kg}}} = 1.76 \text{ m/s} \tag{28}$$

Problem 43. *While running, a person transforms about 0.600 J of chemical energy to mechanical energy per step per kilogram of body mass. If a $m = 60.0$ kg runner transforms energy at a rate of $P = 70.0$ W during a race, how fast is the person running? Assume that a running step is $s = 1.50$ m long.*

This is simply a units conversion problem

$$\frac{70.0 \text{ J/s}}{\text{runner}} \cdot \frac{\text{step kg}}{0.600 \text{ J}} \cdot \frac{\text{runner}}{60.0 \text{ kg}} \cdot \frac{1.50 \text{ m}}{\text{step}} = 2.94 \text{ m/s} \tag{29}$$