

Homework 5

Problem 18. Whenever two Apollo astronauts were on the surface of the Moon, a third astronaut orbited the Moon. Assume the orbit to be circular and $r_1 = 100 \text{ km}$ above the surface of the Moon. At this altitude, the free-fall acceleration is $g = 1.52 \text{ m/s}^2$. The radius of the Moon is $r_0 = 1.70 \cdot 10^6 \text{ m}$. Determine (a) the astronaut's orbital speed v and (b) the period of the orbit.

(a) Using the basic formula for circular motion

$$a_c = \frac{v^2}{r} \quad (1)$$

$$v = \sqrt{a_c r} = \sqrt{g(r_1 + r_0)} = \sqrt{1.52 \text{ m/s}^2 \cdot (1.00 \cdot 10^5 + 1.70 \cdot 10^6) \text{ m}} = 1.65 \text{ km/s} \quad (2)$$

(b) The astronaut travels the circumference at a constant speed so

$$\Delta x = v \Delta t \quad (3)$$

$$T = \frac{2\pi r}{v} = \frac{2\pi r}{\sqrt{a_c r}} = 2\pi \sqrt{\frac{r}{a_c}} = 2\pi \sqrt{\frac{(1.00 \cdot 10^5 + 1.70 \cdot 10^6) \text{ m}}{1.52 \text{ m/s}^2}} = 6.84 \text{ ks} = 1.90 \text{ hrs} \quad (4)$$

Problem 23. A pail of water is rotated in a vertical circle of radius $r = 1.00 \text{ m}$. What is the minimum speed of the pail, upside down at the top of the circle, if no water is to spill out?

At the critical low speed, all of the centerward acceleration comes from gravity (no tension/normal force, like Chapter 5, Problem 47 from recitation). So

$$a_c = \frac{v^2}{r} \quad (5)$$

$$v = \sqrt{a_c r} = \sqrt{g r} = \sqrt{9.8 \text{ m/s}^2 \cdot 1.00 \text{ m}} = 3.13 \text{ U m/s} \quad (6)$$

Just like in Problem 18.

Problem 50. An air puck of mass m_1 is tied to a string and allowed to revolve in a circle of radius R on a frictionless horizontal table. The other end of the string passes through a hole in the center of the table, and a counterweight of mass m_2 is tied to it (Fig. P5.50). The suspended object remains in equilibrium while the puck on the tabletop revolves. What are (a) the tension in the string, (b) the radial force acting on the puck, and (c) the speed of the puck?

(a) Constructing a free body diagram for m_2 , we see that the only forces on it are the tension \mathbf{T} and gravity \mathbf{F}_{g2} . Summing the forces in the downward direction we have

$$\sum F = F_{g2} - T = m_2 g - T \quad (7)$$

$$= m_2 a = 0 \quad (8)$$

$$T = m_2 g \quad (9)$$

Where the first line is summing the forces, the second is Newton's second law, and the third is combining the previous two and solving for tension.

(b) The only radial force acting on the puck is tension so $F_c = T = m_2 g$.

(c) We find the speed of the puck using the circular motion formula

$$a_c = \frac{v^2}{r} \quad (10)$$

$$v = \sqrt{a_c r} = \sqrt{\frac{F_c r}{m_1}} = \sqrt{\frac{m_2}{m_1} g r} \quad (11)$$

Problem 52. An amusement park ride consists of a rotating circular platform $d = 8.00 \text{ m}$ in diameter from which $m = 10 \text{ kg}$ seats are suspended at the end of $l = 2.50 \text{ m}$ massless chains (Fig. P5.52). When the system rotates, the chains make an angle of $\theta = 28.0^\circ$ with the vertical. (a) What is the speed of each seat? (b) Draw a free-body diagram of a $m_c = 40.0 \text{ kg}$ child riding in a seat, and find the tension in the chain.

(a) We will eventually use $v = \sqrt{a_c r}$ as we have in all the other problems in this homework assignment to find v .

First, we need to find the radius r of the path that the seat takes around the ride.

$$r = \frac{d}{2} + l \sin \theta = (4.00 + 2.50 \sin 28.0^\circ) \text{ m} = 5.1736 \dots \text{ m} \quad (12)$$

Now we need to find the centerward acceleration a_c . Drawing a free body diagram of our seat, we see that the only forces acting upon it are the tension \mathbf{T} and gravity \mathbf{F}_g . We know that the seat does not rise or fall in the vertical (y) direction, so summing the forces we have

$$\sum F_y = T \cos \theta - mg = ma_y = 0 \quad (13)$$

$$T = \frac{mg}{\cos \theta} \quad (14)$$

$$\sum F_c = T \sin \theta = mg \tan \theta = ma_c \quad (15)$$

$$a_c = g \tan \theta = 9.8 \text{ m/s}^2 \cdot \tan 28.0^\circ = 5.2108 \dots \text{ m/s}^2 \quad (16)$$

So

$$v = \sqrt{a_c r} = \sqrt{g \tan \theta \cdot \left(\frac{d}{2} + l \sin \theta\right)} = \sqrt{5.2108 \dots \text{ m/s}^2 \cdot 5.1736 \dots \text{ m}} = 5.19 \text{ m/s} \quad (17)$$

(b) Our free body diagram with a child in the seat will be the same as our diagram from (a) but with a new mass $m' = m + m_c = 50 \text{ kg}$.

Before we find the tension in the chain, we should check to see if the chain angle changes. The angular velocity $\omega = v/r$ does not change when people get into the seats (because they are of negligible mass compared to the platform), so we can relate our new velocities v' and r' using the same ω that we had in (a).

$$\omega = \frac{v'}{r'} = \frac{v}{r} = \frac{\sqrt{g r \tan \theta}}{r} = \sqrt{\frac{g \tan \theta}{r}} = 1.00 \text{ rad/s} \quad (18)$$

Not that the numerical value is important, just that it is a constant. We can plug $v' = \omega r'$ into our centerward acceleration equation

$$a'_c = \frac{v'^2}{r'} = \frac{\omega^2 r'^2}{r'} = r' \omega^2 \quad (19)$$

And applying this to eqn. 16 (which hasn't changed except for the need to substitute primed variables)

$$a'_c = g \tan \theta' = r' \omega^2 \quad (20)$$

$$g \tan \theta' = \left(\frac{d}{2} + l \sin \theta'\right) \omega^2 \quad (21)$$

The only unknown in this equation is θ' , but the equation is analytically unsolvable. We know $\theta' = 28.0^\circ$ is one solution, because there are no masses in this equation, so it must also hold for case (a). Then we have to decide if there will be any other solutions. We know intuitively that any solutions will have $0^\circ < \theta' < 90^\circ$. Considering the sin and tan functions on that interval, we see that $\sin \theta'$ is concave down and continuous over the entire interval, and that $\tan \theta'$ is concave up and continuous over the entire interval. Therefore, the left hand side of this equation only equals the right hand side at a single value of θ' so our 28.0° solution is unique. If this doesn't make sense to you, you can graph the right and left hand sides to check.

Having proved that $\theta' = \theta$ we can move on to solve for the tension. Using eqn 14.

$$T' = \frac{m'g}{\cos \theta'} = \frac{50 \text{ kg} \cdot 9.8 \text{ m/s}^2}{\cos 28.0^\circ} = 555 \text{ N} \quad (22)$$

As far as grading is concerned I will accept anything where you did any of the following:

- assumed θ didn't change (skipping the whole $\theta' = \theta$ step)
- assumed ω didn't change, and you went on to show $\theta' = \theta$ is a valid solution (skipping the uniqueness step).
- assumed ω didn't change, and proved that $\theta' = \theta$ is valid and unique.