Homework 4 solutions

Problem 8. *Three forces, given by* $\mathbf{F}_1 = (-2.00\mathbf{i} + 2.00\mathbf{j})$ *N,* $\mathbf{F}_2 = (5.00\mathbf{i} - 3.00\mathbf{j})$ *N, and* $\mathbf{F}_3 = -45.0\mathbf{i}$ *N, act on an object to give it an acceleration of magnitude* $a = 3.75$ *m/s*^{2} *(a) What is the direction of the acceleration? (b) What is the mass of the object? (c) If the object is initially at rest, what is its speed* v *after* $t = 10.0$ *s? (d) What are the velocity components of the object after* $t = 10.0$ *s?*

(a) Summing the forces we have

$$\sum F_x = F_1x + F_2x + F_3x = (-2.00 + 5.00 - 45.0)\ \text{N} = -42.0\ \text{N} \quad (1)$$

$$\sum F_y = F_1y + F_2y + F_3y = (+2.00 - 3.00 + 0)\ \text{N} = -1.00\ \text{N} \quad (2)$$

We know from Newtons second law that

$$\sum \mathbf{F} = m\mathbf{a} \quad (3)$$

So the acceleration $\mathbf{a}$ will be in the same direction as the force $\mathbf{F}$. The direction θ of the force is given by

$$\theta = \arctan\left(\frac{F_y}{F_x}\right) = \arctan\left(\frac{-1}{-42}\right) = (1.36 + 180)^o = 181.36^o \quad (4)$$

Measured counter-clockwise from the $\mathbf{x}$ axis (where we have added 180^o because $F_x < 0$ so we have a backside arctan).

(b) From Newton's second law

$$\sum \mathbf{F} = m\mathbf{a} \quad (5)$$

$$\left|\sum \mathbf{F}\right| = m\,|\mathbf{a}| \quad (6)$$

$$m = \frac{|\sum \mathbf{F}|}{a} = \frac{\sqrt{(-41.0)^2 + (-1.00)^2}\ \text{N}}{3.75\ \text{m/s}^2} = 11.2\ \text{kg} \quad (7)$$

(c) This section is constant acceleration review.

$$v = at + v_0 = 3.75\ \text{m/s}^2 \cdot 10.0\ \text{s} = 37.5\ \text{m/s} \quad (8)$$

(d) Using our velocity $v = |\mathbf{v}|$ from (c) and our angle θ from (a) (we know that $\mathbf{v}$ is in the same direction as $\mathbf{a}$ and $\mathbf{F}$) we have

$$v_x = v\cos\theta = 37.5\ \text{m/s} \cdot \cos 181.36^o = -37.49\ \text{m/s} \quad (9)$$

$$v_y = v\sin\theta = 37.5\ \text{m/s} \cdot \sin 181.36^o = -0.893\ \text{m/s} \quad (10)$$

$$\mathbf{v} = (-37.49\mathbf{i} - 0.893\mathbf{j})\ \text{m/s} \quad (11)$$

Problem 22. *The systems shown in Fig. P4.22 are in equilibrium. If the spring scales are calibrated in newtons, what do they read? (Ignore the masses of the pulleys and strings, and assume that the incline is frictionless.)*

Remember that what a spring scale does is measure the tension pulling on *one* of it's sides when in equilibrium. To see this, imagine a spring scale in it's normal use, hanging from the ceiling with a mass m suspended from it. m is in eqilibrium, so the tension T_1 in the string connecting m to the scale must be $T_1 = mg$. The (massless) scale is also in equilibrium, so the tension T_2 in the string connecting the scale to the ceiling must be $T_2 = T_1 = mg$. The scale has $T_1 = mg$ pulling down and $T_2 = mg$ pulling up, and gives a reading of mg, the weight of the suspended mass.

The text tries to remind you of this somewhat tricky concept on page 108 in quick quiz 4.7.

(a) Starting from the left, the ball of mass $m = 5.00$ kg has two forces acting on it: gravity $F_g = mg$ and tension T_1. Summing forces in the upwards direction we have

$$\sum F = T_1 - F_g = T_1 - mg \quad (12)$$

$$= ma = 0 \quad (13)$$

$$T_1 = mg \quad (14)$$

Where 12 comes from our free-body diagram of the forces on the ball, 13 come from Newton's second law and the fact that the particle is in equilibrium, and 14 comes from combining 12 and 13.

Moving on to the scale, we see that the scale has two forces on it: tension from the left ball T_1 and tension from the right ball T_2. Summing the forces in the rightwards direction we have

$$\sum F = T_2 - T_1 \tag{15}$$
$$= m_s a = 0 \tag{16}$$
$$T_2 = T_1 = mg \tag{17}$$
$$\tag{18}$$

Following exactly the same reasoning we applied to the left ball.

The scale has mg pulling on both sides, so it will read $mg = 5.00 \text{ kg} \cdot 9.8 \text{ m/s}^2 = 49.0 \text{ N}$ (see the note above). (Because of the slight sneaky-ness, I didn't take off if you gave an answer of $2mg$.)

(b) Following the same reasoning we applied to the left ball (eqns 12 to 14) in (a), we have $T_1 = T_2 = mg$ for both hanging masses.

So the pulley has three forces on it: the tensions of the cord connecting the two masses T_1 and T_2, and the tension cord connecting it to the scale T_3. Summing the forces in the upwards direction we have

$$\sum F = T_3 - T_2 - T_1 \tag{19}$$
$$= m_p a = 0 \tag{20}$$
$$T_3 = T_2 + T_1 = 2mg \tag{21}$$

And the scale is in equilibrium, so as in (a) it has T_3 pulling on both sides, and it will read $2mg = 2{\cdot}5.00 \text{ kg}{\cdot}9.8 \text{ m/s}^2 = 98.0 \text{ N}$.

(c) The block has 3 forces acting on it: gravity $F_g = mg$, tension T, and a normal force F_N. Summing forces in the tension direction we have

$$\sum F = T - F_g \cdot \sin\theta = T_1 - mg \tag{22}$$
$$= ma = 0 \tag{23}$$
$$T_1 = mg \cdot \sin\theta \tag{24}$$

And the scale is in equilibrium, so it has T_1 pulling on both sides, and it will read $mg{\cdot}\sin\theta = 5.00 \text{ kg}{\cdot}9.8 \text{ m/s}^2{\cdot}\sin 30^o = 24.5 \text{ N}$.

Problem 24. *Fig. P4.24 shows loads hanging from the ceiling of an elecator that is moving at a constant velocity. Find the tension in each of the three strands of cord supporting each load.*

First, we need to understand the effect of the elevator. It is moving at a constant *velocity* so we know that the acceleration $\mathbf{a}$ of all the elements must be 0. So the elevator's constant motion has no effect on the tensions.

(a) Let $m = 5.00$ kg be the mass of the ball, $\theta_1 = 40.0^o$ be the angle between $\mathbf{T}_1$ and the horizontal, and $theta_2 = 50.0^o$ be the angle between $\mathbf{T}_2$ and the horizontal. Following identical reasoning to Problem 22 (a), we know that the tension $T_3 = mg = 5.00 \text{ kg} \cdot 9.8 \text{ m/s}^2 = 49 \text{ N}$.

Now looking at the knot where the three cords come together. There are three forces acting on the knot: T_1, T_2, and T_3. Letting the upwards direction be $+\mathbf{x}$ and the rightwards direction be $+\mathbf{y}$ we can break our tensions into components

$$T_{1x} = -T_1 \cos\theta_1 \tag{25}$$
$$T_{1y} = T_1 \sin\theta_1 \tag{26}$$
$$T_{2x} = T_2 \cos\theta_2 \tag{27}$$
$$T_{2y} = T_2 \sin\theta_2 \tag{28}$$
$$T_{3x} = 0 \text{ N} \tag{29}$$
$$T_{3y} = -mg \tag{30}$$

Now summing the forces on the knot we have

$$\sum F_x = T_{3x} + T_{2x} + T_{1x} = 0 + T_2 \cos\theta_2 - T_1 \cos\theta_1 \quad (31)$$
$$= m_k a_{kx} = 0 \quad (32)$$
$$T_2 = T_1 \frac{\cos\theta_1}{\cos\theta_2} \quad (33)$$
$$\sum F_y = T_{3y} + T_{2y} + T_{1y} = -mg + T_2 \sin\theta_2 + T_1 \sin\theta_1 \quad (34)$$
$$T_2 = \frac{mg - T_1 \sin\theta_1}{\sin\theta_2} = T_1 \frac{\cos\theta_1}{\cos\theta_2} \quad (35)$$
$$\frac{mg}{\cos\theta_1} - T_1 \frac{sin\theta_1}{\cos\theta_1} = T_1 \frac{\sin\theta_2}{\cos\theta_2} \quad (36)$$
$$T_1(\tan\theta_1 + \tan\theta_2) = \frac{mg}{\cos\theta_1} \quad (37)$$
$$T_1 = \frac{mg}{\cos\theta_1(\tan\theta_1 + \tan\theta_2)} = \frac{49 \text{ N}}{\cos 40^o(\tan 40^o + \tan 50^o)} = 31.5 \text{ N} \quad (38)$$
$$T_2 = T_1 \frac{\cos\theta_1}{\cos\theta_2} = \frac{mg}{\cos\theta_2(\tan\theta_1 + \tan\theta_2)} = \frac{49 \text{ N}}{\cos 50^o(\tan 40^o + \tan 50^o)} = 37.5 \text{ N} \quad (39)$$

(b) The only changes from (a) are $m = 10$ kg, $\theta_1 = 60.0^o$, and $\theta_2 = 0^o$. Plugging the new values into our symbolic equation from (a):

$$T_3 = mg = 10.0 \text{ kg} \cdot 9.8 \text{ m/s}^2 = 98 \text{ N} \quad (40)$$
$$T_1 = \frac{mg}{\cos\theta_1(\tan\theta_1 + \tan\theta_2)} = \frac{98 \text{ N}}{\cos 60^o(\tan 60^o + \tan 0^o)} = 113 \text{ N} \quad (41)$$
$$T_2 = T_1 \frac{\cos\theta_1}{\cos\theta_2} = \frac{mg}{\cos\theta_2(\tan\theta_1 + \tan\theta_2)} = \frac{98 \text{ N}}{\cos 0^o(\tan 60^o + \tan 0^o)} = 56.6 \text{ N} \quad (42)$$

Problem 51. *If you jump from a desktop and land stiff-legged on a concrete floor, you run a significant rist that you will break a leg. To see how that happens, consider the average force stopping your body when you drop from rest from a height of $h = 1.00$ m and stop in a much shorter distance d. Your leg is likely to break at the point where the cross-sectional area of the tibia is smallest. This point is just above the anke, where the cross sectional area of one bone is about $A = 1.60$ cm^2. A bone will fracture when the compressive stress on it exceeds about $\sigma_b = 1.60 \cdot 10^8$ N/m^2. If you land on both legs, the maximum force F_{max} that your ankles can safely exert on the rest of your body is then about*

$$F_{max} = 2F_b = 2\sigma_b A = 5.12 \cdot 10^4 \ N \quad (43)$$

Calculate the minimum stopping distance d that will not result in a broken leg if your mass is $m = 60.0$ kg.

The problem breaks down into two constant-acceleration problems. Call the top of the desk dropping-off-point P_0, the point of maximum velocity when you are just starting to contact the floor P_1, and the point where your shoe soles have compressed a distance d and brought you back to rest P_2.

First consider the constant acceleration portion from P_0 to P_1. Your final velocity v_1 is given by

$$v_1^2 = v_0^2 + 2a_{01}\Delta y_{01} = 2a_{01}\Delta y_{01} \quad (44)$$

Now applying the same equation to the second constant acceleration portion from P_1 to P_2.

$$v_2^2 = v_1^2 + 2a_{12}\Delta y_{12} = 0 \quad (45)$$
$$v_1^2 = -2a_{12}\Delta y_{12} = 2a_{01}\Delta y_{01} \quad (46)$$
$$d = \Delta y_{12} = -\frac{a_{01}}{a_{12}}\Delta y_{01} \quad (47)$$

The acceleration a_{12} is given by Newton's second law (picking down as the $+\mathbf{x}$ direction)

$$\sum F_x = ma_{12x} \quad (48)$$
$$a_{12x} = (\sum F_x)/m = \frac{mg - F_{max}}{m} = g - \frac{F_{max}}{m} \quad (49)$$

(I forgot to include mg in the sum of the forces when I was doing the problem, so I didn't take off points if you forgot it as well.) So

$$d = -\frac{a_{01}}{a_{12}}\Delta y_{01} = -\frac{g}{g - \frac{F_{max}}{m}}h = -\frac{1}{1 - \frac{F_{max}}{mg}}h \tag{50}$$

$$= -\frac{1}{1 - \frac{5.12 \cdot 10^4}{60 \cdot 9.8}} \cdot 1.00 \text{ m} = 1.16 \text{ cm} \tag{51}$$

(Ignoring gravity in your sum of forces, you would have gotten $d = \frac{mg}{F_{max}}h = 1.15$ cm. The correction is very small because $F_{max} \gg mg$.)