Homework 3 solutions

Problem 9. *The mountain lion can jump to a height of $h = 12.0$ ft when leaving the ground at an angle of $\theta = 45.0^o$. With what speed, in SI units, does it leave the ground to make the leap?*

First, we'll convert the height into SI units:

$$h = 12.0 \text{ ft} \left[\frac{1 \text{ m}}{3.28 \text{ ft}} \right] = 3.659 \text{ m} \tag{1}$$

Next, arrange the information we know in a table, calling the launch point P_0 and the peak point P_1.

Point	P_0	P_1
a_x	0 m	
a_y	−9.8 m	
v_x	?	
v_y	?	0 m/s
x	0 m	?
y	0 m	3.66 m
t	0 s	?

Where we know $v_1 = 0$ m/s because P_1 is at the apex of the jump and x_0, y_0, and t_0 through our choice of coordinate frame.

We want to pick an equation to tell us something about the initial velocity (because they told us θ, either v_{x0} or v_{y0} will suffice.). Looking at our 4 constant acceleration equations (text p. 53),

$$v_{xf} = v_{xi} + a_x t \tag{2}$$

$$x_f = x_i + \frac{1}{2}(v_{xf} + v_{xi})t \tag{3}$$

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2 \tag{4}$$

$$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i) \tag{5}$$

We see that eqn. 5 applied to the y direction is perfect, because it has no information in it that we don't already know except v_{y0}.

$$v_{y1}^2 = (0 \text{ m/s})^2 = v_{y0}^2 + 2a_y(y_1 - y_0) \tag{6}$$

$$v_{y0}^2 = -2a_y y_1 \tag{7}$$

$$v_{y0} = \sqrt{-2a_y y_1} \tag{8}$$

And we use the angle to solve for the magnitude of the inital velocity:

$$v_{y0} = v_0 \sin\theta \tag{9}$$

$$v_0 = \frac{v_{y0}}{\sin\theta} = \frac{\sqrt{-2a_y y_1}}{\sin\theta} = \frac{\sqrt{-2 \cdot (-9.8 \text{ m/s}^2) \cdot 3.66 \text{ m}}}{\sin 45^o} = 12.0 \text{ m/s} \tag{10}$$

Problem 19. *A soccer player kicks a rock horizontally off a $h = 40.0$ m high cliff into a pool of water. If the player hears the sound of the splash $\Delta t = 3.00$ s later, what was the initial speed given to the rock? Assume that the speed of sound in air is $v_s = 343$ m/s.*

Let us call the point when the player kicks the ball P_0, the point where the ball lands P_1, and the point where the sound hits the players ear P_2.

Arranging the information we know about the ball in a table,

Point	P_0	P_1
a_x	0 m	
a_y	−9.8 m	
v_x	?	
v_y	0 m/s	?
x	0 m	?
y	40.0 m	0 m
t	0 s	?

Where we know $v_{y0} = 0$ m/s because the ball is kicked horizontally and x_0, y_0, and t_0 through our choice of coordinate frame.

We want to pick an equation to tell us something about the time, (because they told us t_2.). Looking at our 4 constant acceleration equations We see that eqn. 4 applied to the y direction is perfect, because it has no information in it that we don't already know except t_1.

$$y_1 = 0 \text{ m} = y_0 + v_{y0}t_1 + \frac{1}{2}a_y t_1^2 \tag{11}$$

$$\frac{1}{2}a_y t_1^2 = -y_0 \tag{12}$$

$$t_1^2 = \frac{-2y_0}{a_y} \tag{13}$$

$$t_1 = \sqrt{\frac{-2y_0}{a_y}} = \sqrt{\frac{-2 \cdot 40.0 \text{ m}}{-9.8 \text{ m/s}^2}} = 2.86 \text{ s} \tag{14}$$

So using eqn. 4 applied to the x direction, we have

$$x_1 = x_0 + v_{x0}t_1 + \frac{1}{2}a_x t_1^2 = v_{x0}t_1 \tag{15}$$

Now that we know x_1 and y_1, we can find the distance Δx_s that the sound takes returning to P_2. Because it moves in a straight line (more or less), we have

$$\Delta x_s = \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2} = \sqrt{v_{x0}^2 t_1^2 + y_0^2} \tag{16}$$

So the time $(t_2 - t_1)$ taken for the sound to return is

$$\Delta x_s = v_s(t_2 - t_1) \tag{17}$$

And they give us t_2 in the problem, so we just solve this for v_{x0}.

$$\Delta x_s = \sqrt{v_{x0}^2 t_1^2 + y_0^2} = v_s(t_2 - t_1) \tag{18}$$

$$v_{x0}^2 t_1^2 + y_0^2 = [v_s(t_2 - t_1)]^2 \tag{19}$$

$$v_{x0}^2 = \left[[v_s(t_2 - t_1)]^2 - y_0^2 \right] / t_1^2 \tag{20}$$

$$v_{x0} = \sqrt{[v_s(t_2 - t_1)]^2 - y_0^2}/t_1 = \sqrt{[343 \text{ m/s}(3.00 \text{ s} - 2.86 \text{ s}]^2 - (40.0 \text{ m})^2}/2.86 \text{ s} = 9.91 \text{ m/s} \tag{21}$$

Problem 24. *Suppose a copper sleeve of inner radius $r_1 = 2.10$ cm and outer radius $r_2 = 2.20$ cm is to be cast. To eliminate bubbles and give high structural integrity, the centripetal acceleration of each bit of metal should be at least $a_{c\ min} = 100g$. What rate of rotation is required? State the answer in revolutions per minute (rpm).*

For circular motion

$$a_c = v^2/r \tag{22}$$
$$v = \sqrt{a_c r} \tag{23}$$

And if the circular motion is uniform, the time T taken to complete one revolution is given by

$$\Delta x = vT \tag{24}$$
$$T = \Delta x/v = \frac{2\pi r}{v} \tag{25}$$

So the frequency $f = 1/T$ of rotation is

$$f = 1/T = \frac{v}{2\pi r} = \frac{\sqrt{a_c r}}{2\pi r} = \frac{1}{2\pi}\sqrt{\frac{a_c}{r}} \tag{26}$$

Because for a given frequency $a_c \propto r$, we must use the inner radius r_1 to compute the f required to create an acceleration $a_c \geq 100g$.

$$f = \frac{1}{2\pi}\sqrt{\frac{100 * 9.8 \text{ m/s}^2}{0.0210 \text{ m}}} \cdot \frac{60 \text{ s}}{1 \text{ min}} = 2060 \text{ rpm} \tag{27}$$

To reassure ourselves that this frequency creates enough centripetal acceleration a_{c2} at r_2, we can compute a_{c2}.

$$f = \frac{1}{2\pi}\sqrt{\frac{a_c}{r}} \tag{28}$$

$$2\pi f = \sqrt{\frac{a_c}{r}} \tag{29}$$

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The left hand side is constant, so applying this equation to both r_1 and r_2 we get,

$$\sqrt{\frac{a_{c \text{ min}}}{r_1}} = \sqrt{\frac{a_{c2}}{r_2}} \tag{30}$$

$$a_{c2} = a_{c \text{ min}}\frac{r_2}{r_1} = a_{c \text{ min}}\frac{2.20 \text{ cm}}{2.10 \text{ cm}} = 1.05 \cdot a_{c \text{ min}} \tag{31}$$

So the acceleration is indeed $\geq 100g$ throughout the cylinder.

Problem 43. *A ball on the end of a string is whirled around in a horizontal cirlce of radius $r = 0.300$ m. The plane of the circle is $h = 1.20$ m above the ground. The string breaks, and the ball lands $L = 2.00$ m (horizontally) away from the point on the ground directly beneath the ball's location when the string breaks. Find the radial acceleration of the ball during its circular motion.*

The airbourne ball portion of the problem is a 2-D projectile motion problem. Calling the point where the string breaks P_0 and the point where the ball lands P_1, we have

Point	P_0	P_1
a_x	0 m	
a_y	−9.8 m	
v_x	?	
v_y	0 m/s	?
x	0 m	2.00 m
y	1.20 m	0 m
t	0 s	?

We use eqn. 4 applied to the y direction to find t_1

$$y_1 = 0 \text{ m} = y_0 + v_{y0}t_1 + \frac{1}{2}a_yt_1^2 \tag{32}$$

$$\frac{1}{2}a_yt_1^2 = -y_0 \tag{33}$$

$$t_1 = \sqrt{\frac{-2y_0}{a_y}} \tag{34}$$

As we saw in problem 19.

Now applying eqn. 4 to the x direction to find v_x

$$x_1 = x_0 + v_xt_1 + \frac{1}{2}a_xt_1^2 \tag{35}$$

$$vx = x_1/t_1 \tag{36}$$

And plugging these into our circular motion equation

$$a_c = v^2/r = \frac{(x_1/t_1)^2}{r} = \frac{x_1^2 a_y}{-2y_0 r} = \frac{(2.00 \text{ m})^2 \cdot (-9.8 \text{ m/s}^2)}{-2 \cdot 1.20 \text{ m} \cdot 0.300 \text{ m}} = 54.4 \text{ m/s}^2 \tag{37}$$