Homework 1 solutions

Problem 60. *The consumption of natural gas by a company satisfies the empirical equation*

$$V = 1.50t + 0.00800t^2, \tag{1}$$

where V is the volume in millions of cubic feet and t is the time in months. Express this equation in units of cubic feet and seconds. Assign proper units to the coefficients. Assume that a month is 30.0 days.

. Adding units to the equation coefficients:

$$V = 1.50\left[\frac{\text{million ft}^3}{\text{month}}\right]t + 8.00\cdot 10^{-3}\left[\frac{\text{million ft}^3}{\text{month}^2}\right]t^2 \tag{2}$$

We prepare some conversions:

$$1 = \left[\frac{10^6\ \text{ft}^3}{\text{million ft}^3}\right] \tag{3}$$

$$1 = \left[\frac{1\ \text{month}}{30\ \text{days}}\cdot\frac{1\ \text{day}}{24\ \text{hours}}\cdot\frac{1\ \text{hour}}{60\ \text{minutes}}\cdot\frac{1\ \text{minute}}{60\ \text{s}}\right] \tag{4}$$

$$= \left[\frac{1\ \text{month}}{2.592\cdot 10^6\ \text{s}}\right] \tag{5}$$

So converting the units in our equation to ft^3 and s:

$$V = 1.50\left[\frac{\text{million ft}^3}{\text{month}}\cdot\frac{10^6\ \text{ft}^3}{\text{million ft}^3}\cdot\frac{1\ \text{month}}{2.592\cdot 10^6\ \text{s}}\right]t + 8.00\cdot 10^{-3}\left[\frac{\text{million ft}^3}{\text{month}^2}\cdot\frac{10^6\ \text{ft}^3}{\text{million ft}^3}\cdot\left(\frac{1\ \text{month}}{2.592\cdot 10^6\ \text{s}}\right)^2\right]t^2 \tag{6}$$

$$V = 0.579\left[\frac{\text{ft}^3}{\text{s}}\right]t + 1.19\cdot 10^{-9}\left[\frac{\text{ft}^3}{\text{s}^2}\right]t^2 \tag{7}$$

Problem 62. *In physics, it is important to use mathematical approximations. Demonstrate that for small angles ($< 20°$)*

$$\tan\alpha \approx \sin\alpha \approx \alpha = \pi\alpha'/180° \tag{8}$$

where α is in radians and α' is in degrees. Use a calculator to find the largest angle for which $\tan\alpha$ may be approximated by α with an error less than 10.0%.

. To kill both birds with one stone, a table to show the approximations hold and show the % error of the approximation:

α'	α [rad]	$\sin\alpha$	$\tan\alpha$	% error
0°	0.000	0.000	0.000	$\emptyset$
5°	0.087	0.087	0.087	−0.25%
10°	0.175	0.174	0.176	−1.02%
15°	0.262	0.259	0.268	−2.30%
20°	0.349	0.342	0.354	−4.09%
31°	0.541	0.515	0.601	−9.95%
32°	0.599	0.530	0.625	−10.62%

where the % error is given by

$$\%\ \text{error} = \frac{\text{approx.} - \text{actual}}{\text{actual}} = \frac{\alpha - \tan\alpha}{\tan\alpha}. \tag{9}$$

So 31° is the largest whole-degree angle with $< 10\%$ error.