PHYSICS 233: INTRODUCTION TO RELATIVITY
Winter 2016-2017
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MIDTERM EXAM SOLUTIONS

1. Which *two* of the following *best* describe the assumptions that Einstein made when he developed Special Relativity?

(a) The laws of physics are the same in any inertial frame.

(b) Objects in motion remain in motion, while objects at rest remain at rest.

(c) The speed of light is always the same.

(d) Time travel to the past is not possible.

(e) Physical properties of objects must be conserved.

a, c only

2. Which equation could describe how velocities add in Special Relativity? Here, v_{rel} is the relative velocity of a rocket as measured by an observer in the lab frame and v_{proj} is the velocity of an object along the direction of relative motion, as measured in the rocket frame, and v_{lab} is the projectile velocity in the lab frame. All velocities are in units where $c = 1$.

(a) $v_{lab} = (v_{rel} + v_{proj})/(1 + v_{rel}v_{proj})$

(b) $v_{lab} = (v_{rel} + v_{proj})/(1 - v_{rel}v_{proj})$

(c) $v_{lab} = (v_{rel}v_{proj})/(v_{rel} + v_{proj})$

(d) $v_{lab} = (v_{rel}v_{proj})/(v_{rel} - v_{proj})$

a only

3. In some inertial frame, the space interval between two events is $\Delta x = 0$ and the time interval between the two events is $\Delta t > 0$. The shortest time interval that could be measured between those two events is

(a) the proper time

(b) the time interval measured by an inertial observer present at both events

(c) the invariant spacetime interval

(d) zero

a, b, c but not d

4. If two events occur with spatial separation that is perpendicular to the direction of relative motion of two inertial frames (e.g. two events off to the side of Mary's train), then

(a) the spatial separation will be shorter (length contraction) when measured by an observer in a moving frame

(b) the spatial separation is the same for observers in both frames

(c) the time separation between the events is the same for observers in both frames

(d) the time separation between the events is zero in the frame at rest with respect to those events

b only

5. Principle of Relativity

Suppose each of the following is measured in one inertial frame. Will observers in other inertial frames agree with those measurements? In other words, are these quantities observed to be the same from all free-fall frames? Mark "Yes" or "No" for each and *give a detailed explanation* of your answer. No credit will be given for lucky guesses. If the answer depends on the direction of relative motion, discuss why.

(a) The speed of sound.

No. Sounds waves move at the speed of sound relative to the medium in which they travel. If the medium, e.g. air, is in motion with respect to our frame, then the sound speed measured from our frame could be larger (for v_{rel} in the direction of the sound) or smaller (if v_{rel} is opposite the direction of sound and/or because of time dilation).

(b) The density of air inside a tennis ball.

No. The tennis ball will appear squished along the direction of relative motion. The number of molecules inside the ball remains the same, thus a moving frame measures a higher density of air.

(c) The acceleration of an automobile.

No. Measurement of acceleration requires measurement of distance travelled in fixed time intervals. Irrespective of whether there is any length contraction effect, there will always be time dilation, which will differ among the inertial frames.

(d) The number of rotations per second made by a spinning ice skater.

No. Time dilation of the duration of each spin will cause her to spin more slowly as observed from other frames.

(e) Length of a rocket ship

Yes or no, because length contraction depends on the direction of relative motion. If the relative motion is along the nose-tail axis of the rocket, then "No." If the motion is perpendicular to this axis, then "Yes." For all other angles, "No."

6. Trip to Alpha Centauri

NASA wants to send a manned mission to the nearest stellar system, Alpha Centauri, 4 light-years away, to look for signs of inhabitable planets. The astronauts will fly to Alpha Centauri, explore for 1 year, then return. NASA worries about the effects of weightlessness on the astronauts' health and specifies that they should spend no more than 2 years totally free of the Earth's gravity (time as measured by the astronauts), leaving a total of 1 year in the rocket frame for both the outward and inward journey. Your job is to work out the details of the mission.

(a) Assume that we schedule each leg of the journey to last 6 months as measured in the rocket frame. Use the spacetime interval to compute how long each leg of the trip takes as measured on Earth.

In rocket frame, the astronauts don't move, thus $t'^2 - x'^2 = (0.5\,\mathrm{y})^2 - (0)^2 = 0.25\,\mathrm{y}^2$. In the Earth frame $t^2 - x^2 = t^2 - (4\,\mathrm{ly})^2 = 0.25\,\mathrm{y}^2$, so $t = 4.03\,\mathrm{y}$.

(b) To reach Alpha Centauri in 6 months, what must be the velocity of the rocket? Assume that it moves at constant velocity during each leg of the trip.

$v = \Delta x/\Delta t = (4\,\mathrm{ly})/(4.03\,\mathrm{y}) = 0.9923$ That's fast!

(c) How much have the astronauts aged when they return to Earth? Don't forget the time spent exploring!

Exactly 2 years: $t = 2 \times 0.5\,\mathrm{y} + 1\,\mathrm{y}$.

(d) How much have the astronaut's friends and relatives aged while they were gone?

$t = 2 \times 4.03\,\mathrm{y} + 1\,\mathrm{y} = 9.06\,\mathrm{y}$

(e) Suppose the astronauts find a planet with breathable air and lots of water and like it so much that they decide not to return to Earth. Would they still have aged less than their colleagues back on Earth?

Yes, they would age by $4.03 - 0.5 = 3.53\,\mathrm{y}$ less than their Earth-bound friends.

7. Mirror, Mirror

Mary flies in a rocket that contains two pairs of mirrors. One pair has a mirror at the front of the rocket and another at the rear (fore-aft). The other pair of mirrors face each other on the sides (starboard-port). She measures the fore-aft mirrors to be 10 meters apart. She measures the starboard-port mirrors to be 2 meters apart. John observes her flying by and they later compare some measurements. Suppose that the velocity of the rocket as measured by John is $v_{rel} = 0.866$. (In case you don't have a calculator, it may be helpful to know that $(0.866)^2 = 0.75$.) Remember to carefully justify your answers.

(a) Mary measures how much time it takes for light to make a *roundtrip* between each pair of mirrors (e.g., photon goes from front to rear and back to the front). What are these times t'_{fa} and t'_{sp}? (Hint: use units in which $c = 1$.)

A photon will take 10 m each way fore-aft, so $t'_{fa} = 20$ m. A photon will take 2 m each way starboard-port, so $t'_{fa} = 4$ m.

(b) What distance does John measure between the starboard-port mirrors?

2 m, by invariance of transverse dimension.

(c) What time does John measure, t_{sp}, for light to make a roundtrip between the starboard-port mirrors?

Consider a photon emitted by the starboard mirror and then received again at the starboard mirror. In Mary's frame there is no spatial separation between those events. But in John's frame they are separated by $l = v_{rel}t_{sp}$. Use invariance of the spacetime interval:

$$d^2 = (t'_{sp})^2 - (l')^2 = (t_{sp})^2 - (v_relt_{sp})^2 = (4)^2 - (0)^2 = (t_{sp})^2[1 - (0.866)^2] = (t_{sp})^2(0.25)$$

Thus, $t_{sp} = \sqrt{64} = 8$ m.

(d) What time does John measure for light to make a roundtrip between the fore-aft mirrors?

The same time dilation factor affects this "clock" as for the photon bouncing between the starboard-port mirrors. Thus, he observes a longer time by the same factor of 2, thus $t_{fa} = 40$ m of time.

The times change in the same way because time dilation must affect both of the "mirror clocks" in the same way. Thus, in both cases, John measures a time that is twice what Mary measures.