PHYSICS 233: INTRODUCTION TO RELATIVITY
Winter 2018-2019
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Lecture Notes #3: Principles of Relativity
Tuesday, January 29, 2019

0 Preliminaries

Required Reading: *Spacetime Physics*, ch. 3

Homework: HW3 is due this Thursday, January 31. HW4 is due Thursday, February 7.

1 Principles of Relativity

How can you tell if you are moving? What experiment can you do that reveals if you are in motion or at rest? Recall our rocket trip around the Moon.

Like Einstein, Galileo used a thought experiment to develop ideas of relativity. This is, of course, the development of *Galilean* relativity, not Special Relativity. Consider observations below deck on a ship. (DRAW the situation.) The hold is filled with birds, butterflies, other small flying animals. There's a bowl of water with fish in it. Now try jumping up and down or side to side. Take a running jump. If the ship is moving at constant speed (and the waves are not rocking the boat up and down), then there is no experiment that tells you that you're moving. This was the first statement of something like the Principle of Relativity!

Note carefully that this Galilean frame of reference is not a free-float frame. You still have gravity. Galileo obviously wouldn't have thought of our spaceship example!

Einstein's Principle of Relativity: The laws of physics are the same in every free-float (inertial) frame. The theory of special relativity was *derived* from this postulate.

Of course, there are external frames of references that can tell us if we're moving. Looking out the window, we can detect our motion relative to the stars, galaxies, and the cosmic microwave background radiation. We detect our motion with respect to the Cosmic Microwave Background (CMB) by measuring a temperature difference in the forward and trailing direction. In fact, that's how astronomers have determined that our Local Group of galaxies is moving at 600km/s through space. This radiation was emitted when the universe was a mere 300,000 years old and first cooled enough that Hydrogen could form. The temperature was roughly $T = 3000$K then, but the expansion of universe has cooled it to $T = 3$K now. That temperature corresponds to microwave frequency radiation, which is about the same wavelength as used in police radar. Imagine a spaceship with microwave detectors along all three axes – a simple experiment could determine the direction of its motion. But let's consider free-float frames with no windows and deal only with experiments internal to that frame.

To reiterate: in different inertial frames,

the laws of physics are same ($F = ma$, for example)

the constants of nature are same (G, for example),

observables for events may differ, but are related by invariant quantities ($d^2 = (ct)^2 - l^2$, for example)

Thus, another statement of the Principle of Relativity is, No test of the laws of physics can differentiate one free-float frame from another.

Let's discuss Physics and Philosophy for a moment. Einstein developed Special Relativity by *assuming* the Principle of Relativity. The theory works to the accuracy that we can measure, meaning that it makes predictions of phenomena that we can observe in the laboratory and our observations agree with those predictions to within the accuracy that we can measure. Does this mean that this Principle is "true"? A "law" of physics? Part of "reality?" It's "true" to the extent that it does accurately describe reality as we observe it. And it describes reality within a fairly large range of scales: In what realms does this principle really apply?

Not on very small scales – there quantum mechanics shows that space is not really empty – particle-anti-particle pairs, etc. (problems with uniting GR and QM on Planck scale)

Not on very large scales – universe of galaxies, dust, neutrinos, etc., which would keep us from having a patch of empty space in which we could move without crashing into things. And on a large enough scale there's always *something* that causes gravity (it's a force with infinite range!).

So far we've established the properties of free-float frames and stated the Principle of Relativity, that the laws of physics appear identical in all such frames. What's different between frames?

2 Comparing inertial frames

The TIME and SPACE separations can (but don't have to be) different. In general, they're not the same. If the time and space intervals can be different, then the VELOCITIES can be different.

Remember our lattice of rods and clocks? Think of motion of an object within a frame as a sequence of events recorded by our set of high-precision camera-clocks (DRAW path of object as it crosses two different frames.) Suppose it moves along the lattice in the y direction in one frame. In another frame, that moves in the x direction relative to the first frame, the object moves diagonally.

Accelerations and forces are not the same in different frames. Acceleration is the rate of change of velocity. Since the time and space intervals can be different, the accelerations can appear different. The effect of acceleration on an object can in general give it a curved path through the frame - the curves look different in different inertial frames.

Electric and magnetic fields differ between inertial frames; special relativity unites electricity and magnetism into a single theory that shows how the electric and magnetic fields in one frame relate to those in another: electromagnetism (moving charges make a magnetic field!).

The equations of electromagnetism are known as "Maxwell's Equations." A prediction of these equations is that electromagnetic fields propagate as waves with velocity – you guessed – of $c = 3 \times 10^8\,\mathrm{ms}^{-1}$. Electromagnetic waves and light are the same thing. Thought of as particles, photons are the massless particles that mediate the electromagnetic force.

Note that we said that velocities can be different in different inertial frames. This is true EXCEPT for the speed of light. This may seem counterintuitive, but has been verified by many experiments.

Sample problem 3-1: Examples of Relativity. Which of these must be the SAME in all free-float frames?

(a) Speed of light in vacuum
YES. This is an assumption (postulate) of relativity theory that has been experimentally verified.

(b) Speed of an electron
NO. This is clear if we compare speed of electron in its own frame (zero) with that of another observer

(c) Charge of electron
YES. Otherwise you could tell how fast you're moving by measuring electric force between two electrons. ($F = q_1 q_2/(4\pi\epsilon_0 r^2)$)

(d) Kinetic energy of proton
NO. This depends on velocity, which varies with frame ($K = mv^2/2$)

(e) Electric field at a point in space
NO. Try measuring that field by its acceleration on an electron that is in your frame (remember

that electric charge does NOT change). Measured acceleration will depend on frame, thus field must depend on frame. $F = ma$ and $F_{electric} = qE$. Mass, m, and charge q do not change with reference frame, so measuring different acceleration, a, implies you will measure different electric field, E.

(f) Time between events
NO. We've shown that by computing the space and time intervals in different frames from the invariance of the total spacetime interval.

(g) Order of elements in the periodic table
YES. But that's obvious, right? Just depends on number of protons in nucleus. How could that possibly change with velocity?

(h) Newton's first law of motion
YES! That "bodies in motion remain in motion, while bodies at rest stay at rest" is pretty much the def. of a free-float frame

Thursday, January 31, 2019

0 Preliminaries

Required Reading: *Spacetime Physics*, ch. 3

Homework: Problem set 4 is due next Thursday, February 7.

1 Relativity of Simultaneity

Perhaps the strangest part of relativity is that events that occur at the same time in one frame occur at different times in another. How can this be? Let's consider Einstein's "Train Paradox." (DRAW Fig 3-1 on p. 63) Lightning hits the front and rear of a moving train. The lightning strikes leave marks on both the train cars and on the tracks. There are two events: lightning hitting front, back of train.

Observer standing by track sees lightning hit both ends at the same time just as the middle of the train passes by him. He's sure that they're simultaneous because he's in the middle, so the light travel time is identical to both ends.

An **observer riding in the middle of the train** is looking out her window at exactly the same moment (waves to man by railside). What does she observe? Consider this from point of view of man by railside: She is moving towards the front flash and away from the rear flash. Thus, she observes the front flash first because her motion carries her towards it, so the light from the front flash has to travel less distance.

How can this be? Light speed is same in both frames, right? Yes, but the distance travelled by light is different (she moves while light is travelling!).

The woman on the train sees the front flash first. She knows that the speed of light is constant and she knows that she's in the middle of the train. Therefore, she concludes the front flash occurred first. Is she deluded? NO. Relativity tells us that her observations are equally true. But isn't she moving relative to the track, so that we can tell that SHE is the one who is moving? Well, we could have done the same experiment out in empty space, with the woman and man on two spaceships, with meteors hitting the front and rear of her ship.

So, are the events simultaneous or not? It depends on your reference frame!

2 Lorentz contraction (Length Contraction)

We have to be very careful when we think about measuring lengths. Measurement of the correct length of something in our frame of reference requires that we locate both ends at the SAME TIME. Again imagine our lattice of rods and clocks. Use the lattice to mark the position on that lattice of each end of the object when the clocks near those positions read the same time. Remember that our camera-clocks are constantly taking pictures with time stamps on each picture. To measure the length of something, find the simultaneous (same time stamp) pair of images that record the positions of the ends. It's obviously no good to use pictures with different time stamps – the object will have moved!

The relativity of simultaneity implies that observers in different frames can disagree about the lengths of things! If I measure a length using simultaneous measurements, a frame moving relative to mine will say that my measurements were NOT simultaneous "Hey, dummy! The box moved while you were measuring it!"

Now let's return to the train paradox. Lightning left marks on the tracks. The man by the railside saw those marks made at same time, thus he can measure the length of the train using those marks. He records the simultaneous position of the ends of the train car in this frame using the tracks as his ruler. But the woman in the train says, "Hey dummy, those marks weren't made at the same time! The second one was made after the first, and the train had moved! The train is longer than you think!" This is a general result: if we compare the length of an object as measured in its rest frame to its length as measured in a frame in which it is moving, it will appear shorter in the moving frame.

Who is right? What is the correct length? Both are correct measurements. But the length in the rest frame IS special: this is the PROPER LENGTH which is always the longest that one will measure. That's why it's not confusing to call this effect "length contraction," because the length can only appear shorter than this.

IMPORTANT: we can only make sense of the observations if we are careful to make only LOCAL observations. In other words, use the lattice of rods and clocks to record the place and time of each event. To an observer in a given free-float frame, the events are not the places and times as he/she "sees" them – which would include spurious effects due to the time for light to get to him/her. Rather, the events are the places and times as recorded local to the events (use the events recorded by the camera-clocks).

In the Train Paradox example above, we carefully place the woman on the train and the man by the railside in the center of the train. In our example, the man sees the lightning flashes at the same time. He knows that the spots where the lightning hit are both the same distance from him, so that the light-travel time was the same. Thus, he infers that the events took place at the same time. Likewise, the woman sees the flashes at different times. She knows that she is in the center of the train, from which the distances and therefore the light-travel times to both ends are the same. Because the flashes arrive at her at different times, she then infers that the events that caused the flashes must have occured at different times.

There is nothing special about putting the man and woman in the center of the train. We could have put either or both off to one side. Then they would simply need to take the extra step of measuring their distance from the site of each flash to compute the light travel time to each event. It's much simpler to use our system of rods and clocks!

3 Invariance of Transverse Dimension

In our earlier example of Lorentz contraction, we discussed how the man and woman would disagree about the length of the train. An observer moving relative to the train will observe a length for the train that is equal to or smaller than the length measured by the woman on the train.

Now try out the following "Gedankenexperiment":
(DRAW train on tracks at rest, looking at its end.) Trains are specified by their "gauge" – the width between the tracks. For example, many toy trains use HO gauge tracks. The gauge of a train is a standard, so that all trains designed to use this gauge can travel on all tracks built to that specification (which is not the same in different countries). At rest the wheels nestle perfectly on the tracks. What happens as the train speeds up? Consider this from the frame of the moving train: If length contraction does not depend on direction of motion, then from the train's frame, the tracks get narrower and narrower together as the train speeds up! The train falls off the tracks. Oops!

But now consider this in the frame of the tracks: As the train speeds up, the distance between the wheels gets smaller and smaller. The train falls down between the tracks. Recap this argument:

Lengths are smaller when measured in a frame in which the object is in motion.

In train's frame, it is the tracks that are moving.

In track's frame, it is the train that is moving.

This leads to an obvious contradiction! The only logical resolution is that Lorentz contraction simply does not occur transverse to the direction of motion.

Let's try another example: Consider two cylinders of equal diameter. Set them in motion toward each other along their long axis. Does one pass inside the other? Consider this from the frame of each cylinder. It makes no sense that cylinder A passes to the outside in one frame, but cylinder B passes to the outside in another. There IS an experiment you could do to tell which happened!

Define LONGITUDINAL = along the direction of motion, TRANSVERSE = across the direction of motion, exactly perpendicular.

This leads us to a clarification of the effect: Lorentz contraction occurs only in the LONGITUDINAL direction! Therefore, dimensions transverse to the direction of motion will always be the same. We can call this "Invariance of transverse distance"

Similarly, the Relativity of Simultaneity depends on where the events occur with respect to the direction of motion. Let's go back to our Train Paradox example, in which two lightning bolts strike the train. What about events that occur off to the side of the train? Imagine two lightning strikes that hit the left and right side of the train at the same time as observed by the man by the railside. What does the woman in the traincar see? She sees lightning strike outside side windows at same time. Are the events simultaneous in her frame? YES. In general, events that are separated only by transverse distance are simultaneous in both frames.

4 Proof of the Invariance of the Spacetime Interval

Let's show that the invariance of the speed of light implies invariance of the distance interval $d^2 = t^2 - l^2$.

(DRAW Fig 3-4 on p. 68) Setup: Follow the path of light as a flash is emitted toward a mirror, reflects, and returns. The bulb, mirror, and photodetector are all on board a rocket. The rocket travels through a lab.

In the Rocket frame, the light goes straight up and down. In the Lab frame, the light traverses two diagonal paths. Let's be careful to define the events: E=emission of light from bulb, R=reception at photodetector.

(DRAW Fig 3-5 on p. 69) In the Rocket frame, path is up to mirror and back again.
Rocket frame: (DRAW grid of camera-clocks in Rocket)
Distance to mirror = 3m
Light goes up 3 rods, back down 3 rods.
Total distance travelled = 2 x 3m
Space interval between events E and R = 0m
Time interval between events E and R = 6m of time

In the Lab frame, mirror and photodetector move while light is travelling back and forth.

Light travels two diagonal paths to mirror and back to photodetector. (DRAW grid of camera-clocks in Lab frame.)
Mark location of first event E.
Mark location of mirror when light reflects, 3 m up, 4 m over.
Mark location of photodector at event R when light returns, 3 m down, 4 over.
Fill in the diagonal path travelled by the light.
The space interval between events E and R = 8m.
What is the time interval between events E and R – careful! What's travelling along that diagonal path is a flash of light. The speed of light is constant; it travels 1m of space in 1m of time. What's the length of that path? Use the Pythagorean theorem. Each diagonal has length $d^2 = 3^2 + 4^2$, thus $d = 5$ The total length of the path is then 10m. Thus, the time interval between events = 10m.

Now compare the Rocket and Lab frame intervals.
Rocket time interval = 6m
Rocket space interval = 0m
Lab time interval = 10m
Lab space interval = 8m
The space and time intervals are different. But the "spacetime interval" is the same,

$$d^2 = (time)^2 - (space)^2 = 6^2 - 0^2 = 10^2 - 8^2 = 36$$

thus $d = 6\,\mathrm{m}$.

Note the time dilation of 6m in Rocket frame to 10m in Lab frame. And further note that the transverse distance between the mirror and bulb is same in both frames.

What is the speed of the Rocket? As measured by the Lab observer, it moved 8m in 10m of time, thus 4/5 of light speed.

Thus, certain observables, like the space and time intervals individually, differ between frames. But some physical quantities are invariant. The laws of Physics and constants of nature are the same in all free-floating frames. There are certain invariant quantities, like the spacetime interval, that are also independent of frame. Many observed $phenomena$ look different in different frames (for example, electric and magnetic fields).

An important principle is the Invariance of Interval for ALL Free-Float Frames. Understand that the spacetime interval has the same value for ALL POSSIBLE relative velocities of inertial frames.

5 Summary

Relativity of Simultaneity: Two events that occur along the direction of relative motion between two frames cannot be simultaneous in both frames

Lorentz Contraction: An object in motion will be measured to have a shorter length, along the direction of motion, than its proper length, as measured in its rest frame.

Invariance of transverse distances: Dimensions perpendicular to direction of relative motion are the same in all frames.

Transverse simultaneity: Events separated by distance transverse to relative motion can be simultaneous in different frames.

6 Problems/Examples

Use units where $c = 1$!

(1) Can we travel faster than the speed of light? Compare the spacetime intervals in two frames:

$$(t')^2 - (x')^2 = t^2 - x^2$$

Suppose that the primed frame is in a rocket, the unprimed frame in the Lab. The events in question, say two flashes of light, are at the same place in the Rocket frame, thus:

$$(t')^2 = t^2 - x^2$$

The velocity of the Lab frame relative to the Rocket frame is related to the spatial and temporal separations by $x = v_{rel}t$ Substitute above and rearrange a bit:

$$(t')^2 = t^2 - (v_{rel}t)^2 = t^2[1 - v_{rel}^2]$$

thus

$$t' = t\sqrt{1 - v_{rel}^2}$$

So, if the Lab measures t, the Rocket observer measures t'

Crank up the speedometer and see what happens! Ride in the Rocket and measure the time between the flashes. Remememember v_{rel} is in units of the speed of light. As v approaches c, the time measured in the Rocket frame gets arbitrarily small ($t' = 0$ when $v_{rel} = 1!$). But the velocity in lightspeed units can't get larger than 1. Why? This would require the Rocket clock to read imaginary time. Where's that dial on the clock?

(2) Does a moving clock really run slow?

What's real? Is the time between ticks of the clock on a Rocket really smaller than the time between ticks back on Earth? Doesn't this mean that you can change "reality" by changing your velocity? Hmmm. What do you mean by "reality?" The change of clock rates as a function of speed has been experimentally verified to extremely high accuracy. As you speed up, your clock runs slower relative to clocks back on Earth. Does speeding up affect the mechanics of the clock? ALL measurements of time in your frame, including the rate of molecular reactions in your body, which cause ageing, are the same. It's all consistent within your inertial frame. Certainly looks real enough. Go back to Earth. You're younger than your twin brother. Is that real enough?

STUDENTS: READ AND THINK ABOUT THIS!