PHYSICS 233: INTRODUCTION TO RELATIVITY
Winter 2018-2019
Prof. Michael S. Vogeley

Lecture Notes #2: Floating Free
Lecture 3, January 15, 2019

0 Preliminaries

Required Reading: *Spacetime Physics*, ch. 2

HW: HW1 due on Thursday. HW2 will be assigned on Thursday and will be due one week later.

Topics for the next few lectures:

- Freedom from gravity's clutches - free-floating (inertial) frames
- Definition and properties of inertial frames
- Rods and clocks
- Velocity in units of c
- An infinite number of inertial frames
- Events are the real objects of Physics

1 Freely-Falling Frames: Trip to the Moon

First, please note that the following terms are equivalent: Inertial = free-floating = Lorentz frame. I'll try to remember to follow the book and call it a free-floating frame. There is no relative acceleration between objects in a free-floating frame!

In Jules Verne's story, "A Trip Around the Moon," he makes an important mistake. (DRAW the whole flight path.) A rocket ship (really, a "cannon ship," since it has no rockets on board) is shot up toward the Moon out of a cannon. There is an initial large acceleration, after which the ship is freely-floating all the way to the Moon. There are people inside – where do they stand? Initially they are plastered to the floor. Once in space, Verne thought they would stand first on the side toward Earth, then on the side toward the Moon.

Now imagine a space dog outside the ship (poor pup!). He floats freely alongside the rocket, right? Verne was obviously wrong; what's special about the inside/outside of the rocket?

Time for some thought experiments: If there were no windows, how would you tell when you were

fired from the cannon?
: Slammed back into your seat – short, extreme acceleration

still within Earth's atmosphere?
: Fall towards nose – air resistance slows the rocket (this only applies in case of cannon, not a rocket that continues its thrust)

leaving Earth's atmosphere?
: Floating free!

For the inside of the rocket to be an inertial frame, it is important that there are no thrusters or other means of accelerating the rocket. Now, do you notice when you are

entering orbit around the Moon?
: Don't notice!

orbiting the Moon?
: Don't notice!

leaving Moon's orbit?
: Don't notice!

re-entering Earth's atmosphere?
: Fall towards nose – Air resistance

landing on the Earth again?
: Ouch!

Could you tell (remember, no windows) if the rocket were

spinning on its long axis? Yes.

tumbling head-over-heels? Yes.

traveling tail first instead of nose first? Only when it re-enters atmosphere.

What do we observe if there are multiple spacecraft fired toward the moon? For example, what if there were two spacecraft shot up, one following after the other along an identical path? (DRAW the paths.) In this case, we see the effects of stretching of spacetime along path, due to gravity (General Relativity shows that gravity is just curvature of spacetime). They follow the same path around Moon. As they approach the moon, SC1 speeds up, pulling away from SC2. As it nears then Moon, SC2 starts to catch up. Leaving the Moon, SC1 slows as it leaves, then SC2 slows. Thus, the distance between them increases on the way in toward the Moon, but decreases back to the original separation as the spacecraft leave the Moon. This change in distance between them occurs even though neither ship fires its engines.

Now, what if two spacecraft are shot up at the same time, one a few hundred yards off to one side from the other, both on trajectories that return to Earth? (DRAW the paths.) Now we see the effects of stretching of spacetime *across* their path. These spacecraft follow *different* paths around the Moon. In fact, for both spacecraft to be on trajectories that return them to Earth, they cross paths on the opposite side of the Moon!

For our next thought experiment, imagine what happens if there were three spacecraft shot up at the same time with the same thrust, with the two extras a few hundred yards above and below the first. (DRAW the paths.) They are all send off at the same velocity, with the central one on exactly the right path to bring it back to Earth. They follow *different* paths around Moon. The middle one returns to Earth. The top one might fly off into space. The bottom one might crash into Moon, or might miss Earth on its return. Exactly what happens depends on the strength of the tidal force, which depends on the distance from the Moon at closest approach.

2 Inertial frame inside the ship

We've seen what happens to spacecraft that are on slightly different trajectories on their trips to the Moon. The cause of those differences in paths – the curvature of spacetime – will govern the behavior of objects *inside* a spaceship.

Let's repeat the observations of our space traveler above, this time with a few rubber balls along for the ride.

At blast off: balls hit floor with traveler.

In atmosphere: balls fall toward ceiling.

Out of atmosphere: float freely next to traveler.

Now, while on the way to the Moon, arrange a circle of balls hanging in the air in front of you, with the center of the circle at exactly the center of mass of the ship. What happens as we approach and enter orbit around Moon? You will notice relative motions of balls due to their relative acceleration. This relative acceleration is caused by the different magnitude and/or direction of "force of gravity" felt by each ball. In GR, we say that this is due to the different curve of spacetime. What happens to the circle? A distortion of the circle occurs because each ball is attracted to the center of the moon.

Balls nearer/further from the moon feel a larger/smaller attraction, so there is a net acceleration between the top and bottom ball. Thus the circle is stretched into an ellipse.

Balls off to either side have a small component of force that pushes them toward the center of the circle. Thus, the right and left edges of the circle squish in, further distorting the circle into an ellipse.

Also note that the passenger exits "upside down" (as does the whole ship).

One more time: repeat this experiment with a SPHERE of balls. What happens to the sphere?

Sphere is stretched up and down into an ellipsoid as it nears Moon because the near side feels a larger force.

More vertical stretching as it leaves Moon.

Stretched front to back as it approaches Moon. Lead balls speed up, rocket and following balls catch up.

Compressed back into original length as it leaves Moon Lead balls slow down, then rocket, following balls slow down

Note that the last result MUST be true because the lead and following balls are on same curve through space, so they must arrive back at Earth with same separation. This is not true for balls to either side or above/below, which experience net accelerations relative to the center of mass of the ship and are now on different paths through space.

What would happen if all the bolts on the rocket were undone while freely floating toward the Moon? Would the rocket fall apart? It could! All the pieces and parts would float along until it started to feel stresses due to relative acceleration differences due to Moon. In orbit around Moon, pieces could get all scrambled up! Don't count on re-assembling the rocket for the return journey.

These experiments show that the no inertial frame exists for the whole flight path around the Moon. But for short enough lengths of time, you would not and could not notice these relative accelerations and could define a perfectly could freely floating frame around you. We'll discuss in a moment just how large a region of spacetime can be considered as "freely floating" or "inertial."

3 More examples of freely-floating frames

Let's further explore the idea of freely falling vs. accelerated frames with more objects and people in motion. (DRAW some pictures.)

Example from the book: ball thrown within a house. Cut the house loose from the cliff!

Basketball court on stilts: While Lebron James is is in the air, cut the stilts!

Now consider a more dangerous example: The Professor jumping out the window. Wait - there are no windows in this room. Oh well, how about just off the desk... DEMONSTRATION:

Jump sideways with a ball.

Jump sideways with two balls.

Jump off the desk throwing a ball.

Jump off the desk throwing two balls.

Again, air provides some resistance, but otherwise a free-floating frame.

Athletes use this notion of a free-floating frame all the time. For example,

Skateboarder or snowboarder flies off a jump, spins the board beneath him, boards lands under his feet.

Lebron James arches toward the basket clinging to the ball with just his fingertips – he has big hands, but barely needs to cradle the ball to guide it into the basket.

Newton's First Law, the "Law of inertia": Bodies at rest stay at rest, bodies in motion stay in motion. RELATIVE TO THAT INERTIAL FRAME OR ANY OTHER INERTIAL FRAME!

Why focus so much as gravitational acceleration? We know how to "turn off" many other forces (turn off the rocket engines, decrease the pressure of air), but gravity is almost always there. There are not too many laboratories out beyond the Sun.

4 The Vomit Comet

To do experiments in a free-floating frame, scientists use drop towers, airplanes, and even the Space Station.

NASA's microgravity program uses both a drop tower and an airplane. I know of this through my friend Gareth McKinley at MIT, who has a research grant to work with NASA to study fluid physics in microgravity.

On the ground they have a drop tower in which it takes an object 2.2 seconds to reach the bottom. (DRAW this.) You put your experiment in a box, drop the experiment down the shaft, and carefully catch it at the bottom. In free fall, the experiment feels an acceleration of less than $10^{-4}g$, where $g = 9.80\,\mathrm{m\,s^{-2}}$ is the gravitational acceleration at sea level on Earth.

More fun is an airplane used to simulate free fall. It's a KC-135 nicknamed, appropriately enough, the "Vomit Comet". (DRAW the flight path.) It flies a parabolic trajectory that produces freefall at the top. For a full 20-25 seconds, the inside of the plane is a freely falling frame. The goal is acceleration of less than 0.01G in all three axes, with relative acceleration much less than that. [See notes from zeta.lerc.nasa.gov/kjenks/kc-135.htm for details of flight path] This plane was used to film scenes in the movie "Apollo 13" with Tom Hanks. Several Drexel engineering students flew in it a couple of years ago.

Longer duration freefall experiments can be done in the Space Station but these are expensive and one needs to flight test every component.

Physics review

This is not a course in classical mechanics. I expect that you are at least familiar with the following equations. See me immediately for extra help if these are not familiar. The position of an object moving in one dimension that begins at initial position x_0, moves at initial velocity v_0, and accelerates at a is

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

Newton's force law, which relates the force F of an object to its mass m and its acceleration a is

$$F = ma$$

Newton's gravitational force law

$$F = -\frac{GMm}{r^2}$$

tells us the force on an object with mass m in the gravitational field of an object with mass M that lies a distance r away. The minus sign is because the force is toward M, thus trying to reduce r. Combining the latter two, the gravitational acceleration on m is then

$$a = F/m = -\frac{GM}{r^2}$$

where G is the gravitational constant (look in the back of the book).

Lecture 4, Thursday, January 17, 2019

0 Preliminaries

Required Reading: *Spacetime Physics*, ch. 2

HW: HW1 is due TODAY. HW2 is due Thursday, January 24. Do the following problems from Chapter 2: 4, 9, 10, 11, 13. (Again, answers to odd-number problems are in the back of the book – you must provide solutions that lead to these answers. Where answers are either "yes" or "no" you must justify your answer).

1 MORE ON INERTIAL FRAMES

This classroom is not a good inertial frame. Why? Watch the ball. Set it flying and it curves. Why? Newton would say because of the force of gravity pulling the ball toward the Earth. Einstein would say because the Earth is pushing us up and keeping us from following the curve of spacetime.

While we're falling, we feel no acceleration. It's the acceleration at the bottom that hurts!

All around us we're applying forces to keep objects from following their natural spacetime curve. We're almost always falling!

You hold up your pen.

Chair holds you up (why don't you fall to the center of the Earth?)

Floor holds up chair

Basement holds up floor

Ground holds up basement

The rocket ship examples that we considered on Tuesday showed that a frame falling with gravity is just going with the flow, following the path of least resistance through curved spacetime.

2 EQUIVALENCE OF INERTIAL AND GRAVITATIONAL MASS

Does everything fall at the same rate? Why or why not?

In an inertial frame, the paths taken are the same, or nearly the same, for all objects in the frame. A fundamental concept that is consistent with gravity as curvature is the *equivalence of inertial and gravitational mass*. In Newtonian gravity $F = ma$ and $F = -GMm/r^2$. Same m! That means that $a = -GM/r^2$, regardless of composition of m. But we need to verify this! Recall the story about Galileo dropping balls off the leaning tower of Pisa that had different composition: gold, lead, copper, wood, and stone (by the way, there is strong evidence suggesting that Galileo never did this experiment).

A more advanced test was the Eötvös experiment - 1905. This compared Earth's gravitational acceleration on wood vs. platinum. It showed that acceleration of Earth is same within 1/(100 million) $= 10^{-8}$. More recent was the Dicke et al. experiment at Princeton, actually a series of experiments in the 1960's. They compared the Sun's acceleration on aluminum vs. gold. The gravitational accelerations were the same to 1/(100,000 million) $= 10^{-11}$ The most recent experiments, at the University of Washington (the Eöt-Wash Group), show that the accelerations are the same to 10^{-13}.

3 TEST PARTICLES

Since all objects fall the same, we can imagine "test particles" that probe the structure of spacetime. As an example, imagine space filled with dust. Each particle of dust is so small that it rarely bumps into another particle. Each is so light that it causes no measurable curvature of spacetime.

In most circumstances, we could use much more massive objects as test particles. The important thing is that the gravitational attraction of the particles for each other should be infinitesimal compared to the effects of larger masses, like the Earth, for instance. In general, the gravitational effect of test masses must be negligible.

In our rocket ship example, the rocket is so much less massive than the Earth and Moon that we could consider the gravitational field as being caused solely by the Earth and Moon. To be precise, we know that the orbit of rockets around Moon occurs around the center of mass of the Moon-Rocket system. But the rocket is so light that we can safely neglect the rocket's mass.

(Question: Why isn't the Earth attracted to YOU? Well, actually, it is!)

Thinking about gravity as curvature of spacetime, the curvature caused by the test particles must be undetectable, so that the curvature of space does not change during our experiments. To observe the curvature of spacetime, we could watch this sea of dust, just like we observed the balls in the rocket.

Following is a mathematical expression, using Newton's law of gravity, of what makes a "test particle:" Consider a very large mass, M, and a smaller mass, m. The force of gravity is

$$F = -\frac{GMm}{r^2}$$

Thus the acceleration on the small mass is

$$a_m = -\frac{GM}{r^2}$$

and similarly for a_M, but swapping little m for big M. It's obvious that for $m \ll M$, the acceleration on m is much smaller than the acceleration on M, $a_m \ll a_M$, so we make the approximation that only the small mass moves under the force of gravity.

4 INERTIAL FRAME = LOCALLY FLAT SPACETIME

Think of gravity as the bending of spacetime. An inertial frame is one in which spacetime is flat. No region of space is perfectly flat; there's always mass and energy somewhere that is causing spacetime to warp. But we can find locally flat regions of spacetime, or at least flat enough that we can't detect the effects of gravity.

Try this analogy: the Earth is round, but it's flat enough that we don't consider its curvature when we survey property or build a building (how can you tell that the Earth is NOT flat?). We use a bubble level to indicate "down," which we assume to be the same direction everywhere on the site. Actually, this is an excellent analogy, because the center of curvature of the Earth is in the same place as the center of the gravitational "force" that we feel.

5 MOTION OF MASS-ENERGY CAUSES VARIATION OF GRAVITATIONAL FIELD

What causes the tides? How many are there per day?

I mentioned that we want test particles that do not themselves alter the curvature of spacetime. But it's important to point out that, over long enough periods of time, the curvature does change! These curves in spacetime change as the mass-energy moves around. Think of the tides: these are due to a change in the curve of spacetime as Moon orbits Earth!

Think of the water on the Earth's surface as a set of test particles, but this time with pressure – water is nearly incompressible. (DRAW tides on Earth as if water were a set of test particles.) There are two tides per day as water follows the Moon. There is also the gravitational effect of the Sun on this water.

The magnitude of tide depends on lots of things. (DRAW Earth-Moon-Sun, include tilt of Earth.) Note that the Moon is in the ecliptic plane with Sun, inclined 23.5 deg to the Earth's axis of rotation. The tides change with

Position of Moon

Proximity of Moon – Earth-Moon distance varies by 10% over time

Season – tilt of Earth and proximity to Sun

The tides are also affected by the local topography of the ocean floor. Large tides are caused by water in enclosed regions and resonance. For example, the Coast of Maine and the Bay of Fundy in Nova Scotia (I've been there!) have large tides – up to 16.6 meters. The Mediterranean Sea has very small tides because water doesn't easily flow out of the Straits of Gibraltar. This fooled early scientists, many of whom lived near this sea, into falsely concluding that the Moon did not have a noticeable gravitational effect. The magnitude of tides can also be affected by resonance between the natural oscillation frequency of water sloshing around in that region and the diurnal tides caused by the Moon (remember sloshing the water around in the bathtub until it started to overflow?).

How large is the effect of Moon's orbit at sea level?

$$\begin{aligned} a &= a_{Earth} + a_{Moon} \\ a_{Earth} &= g = -GM_{Earth}/R_E^2 = -9.80\,\mathrm{m\,s^{-2}} \\ a_{Moon-Max} &= +GM_{Moon}/(r_M - R_E)^2 \\ a_{Moon-Min} &= +GM_{Moon}/(r_M + R_E)^2 \end{aligned}$$

(plus sign is because acceleration is away from Earth, toward Moon) where

$$\begin{aligned} G &= 6.673 \times 10^{-11} \frac{\mathrm{m}^3}{\mathrm{kg\,s}^2} \\ R_E &= 6.378 \times 10^6\,\mathrm{m} \\ r_M &\approx 3.84 \times 10^8\,\mathrm{m} \end{aligned}$$

Thus, distance from water to Moon varies by 3.3%.

$$\begin{aligned} M_E &= 6 \times 10^{24}\,\mathrm{kg} \\ M_M &= 0.0123 M_E \end{aligned}$$

Thus, Moon is 1.23% of the mass of the Earth. Variation in acceleration from Moon is ($G = 6.67 \times 10^{-11}$)

$$a_{Moon-Max} - a_{Moon-Min} = 1.8 \times 10^{-4}\,\mathrm{m\,s^{-2}} = 1.8 \times 10^{-5} g$$

Thus, tides are due to a variation in gravitational acceleration of 2 parts in 100,000. Not enough for you to notice, but enough to have influenced, we think, the evolution of life on Earth! (Why might that be? Ideas?)

Ordinarily, we can think of motions of mass-energy as simply changing the curvature of spacetime. But when masses are extremely large and the motions of the bodies fast enough, they can produce waves in space – "gravitational waves" or "gravitational radiation". We'll look at this again in the last week of the course.

6 SIZE OF FREELY-FLOATING = INERTIAL FRAMES

Let's look at limits on the size of a free-floating frame. The rocket examples on Tuesday showed that, given enough time, one can observe relative accelerations of objects. All inertial frames are LOCAL. An inertial frame is a region of space and a duration in time when effects of gravity or other acceleration are negligible. In other words, the size of a free-floating frame is determined by our ability to detect differences in the spacetime paths taken by objects in that frame. Note that differences in the paths are caused by *tidal* forces, i.e., *non-uniform* gravity.

When we looked at multiple rockets, we thought about those rockets as "test particles" in spacetime. Nearly all of the gravitational force or bending of spacetime comes from Moon and Earth, not the rockets themselves. Given enough time and very sensitive measurement devices, we would detect relative accelerations between the rockets, but we could pick a very small period of time during which we would not notice that the rockets were on different paths.

Likewise, the balls inside the rocket were test particles following different paths in spacetime. We saw that, over the course of the rocket's trip around the Moon and back, you would see the balls move. We can think of their motion as being caused by a relative acceleration, due to the different pull of the Moon on each ball. In relativity, we think of each ball as following it's own path through spacetime, along a curve in which it experiences no force at all.

Let's see how the size (in both space and time) of an inertial frame depends on how accurately we can measure. Try this example of a rocket with balls orbiting Moon: The balls are 1m apart, one above the other, in orbit 10km above surface of Moon, which has radius $R_M = 1.738 \times 10^6\,\mathrm{m}$ (DRAW diagram). The relative acceleration of the balls is

$$\delta a = GM/(R_M + 10\,\mathrm{km})^2 - GM/(R_M + 10\,\mathrm{km} + 1\,\mathrm{m})^2 = 1.8 \times 10^{-6}\,\mathrm{m\,s^{-2}}$$

The distance traveled in time t at constant acceleration a and with initial velocity $v_0 = 0$ is

$$x = \frac{at^2}{2}$$

How large is the frame if we could detect $x = 1\mu\,\mathrm{m} = 10^{-6}\,\mathrm{m}$? You get $1\mu\,\mathrm{m}$ of motion in $1.054\,\mathrm{s} = 3.161 \times 10^8\,\mathrm{m}$ of time (solve for $t = \sqrt{2x/a}$). Suppose that we can measure distances to 1mm. Then we get 1mm of motion in $33.3\,\mathrm{s} = 10^{10}\,\mathrm{m}$ of time. A freely-falling frame is a 4-dimensional volume of spacetime $1\,\mathrm{m} \times 1\,\mathrm{m} \times 1\,\mathrm{m} \times (10^{10}\,\mathrm{m}) = 10^{10}\,\mathrm{m}^4$.

Recall the example of the Vomit Comet: Spatial dimension is about 1m x 1m x 1m x 20 seconds. 20 seconds $= 6.0 \times 10^9\,\mathrm{m}$ of time. This is a long thin piece of spacetime, with length determined by the flight of the plane. We start to notice accelerations as the plane pulls out of free-fall.

Summary: Characteristics of free-float frame

- get rid of gravity
- in Newton's language, depends on equal acceleration of all particles at a given location
- every inertial frame is of limited extent in spacetime

7 RODS AND CLOCKS - MEASUREMENTS IN A FREELY-FLOATING FRAME

How does the observer measure the location and time of events in his frame? Think back to our example of Mary driving the rocket through the lab while John watches. How did John measure the location and time of each event? How did Mary measure the location and time of each event?

The challenge is how to build a measuring system in an inertial frame. We use lots of rods and clocks! Start a pile of 1 meter rods and lots of clocks. Assemble a lattice of rods with a clock at each vertex. Now synchronize the clocks. How do we synchronize? Start with a reference clock and all other clocks set to 0. Measure the distance from the reference clock to each vertex clock. Compute the time for light to travel from the reference clock to each vertex clock and add that many seconds to each clock. Now flash a signal from the reference clocking that starts all the others. For example, a vertex clock that lies 10 seconds of distance away begins with 10 seconds of time on it. When it sees the flash, it starts ticking. In the 10 seconds it took light to get from the reference clock to that vertex clock, 10 seconds elapsed on the reference clock, so they are now synchronized.

Events are recorded by the grid of clocks. Each clock has a camera that imprints the time of events on the picture. Imagine each "camera clock" as a video camera the imprints the time on every frame of the film. Observations of events are made by specifying the location on the grid of meter rods and the time on the nearest vertex clock. This may seem artificial, to use clocks spaced by one meter. No problem – we could make an arbitrarily fine grid of rods and clocks.

How do we test if you're in an inertial frame? Set some test particles loose in the system of rods and clocks. Particles initially at rest should stay that way. Particles in motion should continue at the same velocity, in same direction. Use the lattice of rods and clocks to look for motions that are not straight lines!

Note the equivalence/interchangeability of frames. There is no "correct" frame. Lab and rocket are just labels – neither is special.

What about events/objects that are larger than a single inertial frame? Solve this by piecing together a sequence of small inertial frames. Effectively, that's what General Relativity does.

Note again that we measure particle speed in units of the speed of light. The speed of light c is a constant, so if we measure time and distance in the same units, be they meters and light-meters or years and light-years, the velocity in units of c will be the same!

Some example problems:

2-1 Human Cannonball: Person in an elevator is shot out of a cannon (large cannon or very small elevator!). Neglect air resistance.

(a) What happens if the person jumps while elevator is on way up? Do they fall back to the floor, hit the ceiling, or something else?
Ans: They hit the ceiling.

(b) The person jumps on the way back down. What happens?
Ans: They hit the ceiling.

(c) How can a person tell if the elevator has reached the top of its trajectory?
Ans: They can't!

2-5 Earth's Surface as a Free-Float Frame

(a) A particle flies with $v = 0.96c$ through a spark chamber 1m wide. How long to travel 1m? Compare to the motion of a test particle released from rest in this time. Compare to the diameter of an atomic nucleus, a few times $10^{-15}\,\mathrm{m}$.

Assume the particle flies horizontally. At the same time, it is falling downwards because of gravity (which causes acceleration of $g = 9.80$m/s^2 near the surface of the Earth). At $v = 0.96c$, the particle travels 1m sideways in time $t = 1/0.96 = 1.04\,\mathrm{m}$ of time or $3.47 \times 10^{-9}\,\mathrm{s}$. In that time, the gravitational force on the particle causes it to drop a distance $y = gt^2/2 = 5.9 \times 10^{-17}\,\mathrm{m}$, which is smaller than an atomic nucleus, which has characteristic size $\sim 10^{-15}$m.

(b) How long can the chamber be for it to be considered a freely-floating frame for this experiment? Suppose we can measure to one wavelength of blue light, $5 \times 10^{-7}\,\mathrm{m}$.

A particle will fall this distance from rest in time $t = \sqrt{2d/g} = 3.19 \times 10^{-4}\,\mathrm{s}$. A particle at 0.96c will travel $x = vt = (0.96)(3.0 \times 10^8 m/s)(3.19 \times 10^{-4} s) = 9.2 \times 10^4\,\mathrm{m}$, about 90km, in this time.