PHYSICS 233: INTRODUCTION TO RELATIVITY
Winter 2018-2019
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Lecture Notes #1: Einstein and the Unity of Space-Time
Lecture 1: Tuesday, January 8, 2018

0 Preliminaries

Syllabus: Hand out and review.

Required Reading: *Spacetime Physics*, ch. 1

Suggested Reading: *Subtle is the Lord...*, ch. 6

Homework: HW1 is due Thursday, January 17 at the beginning of class. The problems are in Ch. 1, problems 4(a,b,c only), 5, 8, 11, 12. For problem 4, be sure to write out the math that led you to the answers. Do not simply fill in the table.

Note: the solutions to odd-numbered problems are in the back of the book. You get no credit for simply copying those answers! You must always show how you solved a problem.

Important Concepts:

- Einstein
- Postulates of SR
- Implications of SR
- Distance Metric (Surveyor's parable)

1 Albert Einstein

In the Fall of 1999, Albert Einstein was named Time Magazine's Person of the Century. This award was given to "the man or woman who most influenced our times." In a century that was marked by tremendous political, philosophical, and cultural upheavals, it is remarkable that a physicist should be singled out in this way. Why was Einstein selected? A quick review of his work reveals the breadth and depth of his influence on our understanding of the physical world:

Statistical Physics – underpinnings of thermodynamics, solid state physics.

Special Relativity – unity of space and time.

General Relativity – bending of space and time by mass/energy = gravity.

Quantum Theory – light as quanta of energy, sub-atomic interactions of particles.

Einstein spent much of his later career trying to find a physical theory that would unify gravity and quantum theory. That quest continues to this day.

AE was Born in Ulm, Germany in 1879. Contrary to legend, he was not a poor student. Perhaps inattentive at times, but certainly not slow. Through age 15 he earned the highest grades in his class in Math and Latin. From age 12 through 16 he learned differential and integral calculus by himself. What is true is that he disliked classes and preferred to study by himself.

It is true that he had a hard time getting his first job. When he had completed 4 years of study in Switzerland, he was qualified to be a Physics teacher, roughly the equivalent of a university professor, but he could not find such a position. He taught as a substitute high school teacher, then at a private school. Eventually, at the age of 23, he began a job in a patent office. Not exactly his dream job, but it left him spare time and energy to pursue his true love: Physics.

What do you think of when you think of Switzerland? Clocks and chocolate? Well, the clocks are important for our story. Also trains. An important technological problem at the time was how to synchronize clocks at railway stations around the continent, so that trains could be kept on schedule. We imagine that, in his role in the patent office, Einstein saw lots of ideas for solving this synchronization problem. Which got him to thinking...But we're getting ahead of our story.

In 1905 (his $annus\ mirabilis$), he published four formidable papers. Any one of these would have been sufficient to ensure that he would be remembered as one of the greatest physicists of his time. But all three?!

- Quantum theory of light (the photoelectric effect)
- Brownian motion explained in terms of random collisions of molecules
- Special relativity
- Mass-energy equivalance $E = mc^2$

Believe it or not, Einstein did NOT win the Nobel Prize for relativity! It was still considered somewhat controversial at the time he won the prize (for his work on the photoelectric effect), in 1922. Then Millikan won for trying (and failing) to prove him wrong!

If you'd like to walk in his footsteps, literally, you need only travel up the road to Princeton, NJ, where he lived at 112 Mercer Street, a small white house distinguished only by the fact that he lived there. And you could stroll along the quiet streets near the Institute for Advanced Study where he worked for the last 22 years of his life. But there's no grave site. At his request, his ashes were strewn by his family in an unknown location (maybe in Trenton, NJ), though his brain was removed and preserved by a pathologist at Princeton Medical Center (the tale of what happened to his brain is quite a whopper!). Once thought unremarkable, it turns out that his brain was somewhat unusual in its structure. Maybe the hardware is important after all.

[See recommended biographies for more.]

1.1 Political/cultural impact

His work shows the tremendous impact of science on our world view, of technology on our way of life. He was a leader in developing the physics that underlies many of the technical advances of the 20th century.

$E = mc^2$
How many other equations do people know of? That equation leads to the possibility of turning mass into energy: In 1939, six years after coming to the United States to become one of the first faculty at the Institute for Advanced Study in Princeton, he wrote a letter to President F.D. Roosevelt, alerting him to the possibility of constructing an atomic bomb. Though he did not work on the bomb himself and is by no means the father of the atomic bomb, his early work paved the way for the physical insight necessary to create this fearsome weapon of destruction.

"Relativity"
Observer-dependent nature of space vs. time. These ideas have been culturally interpreted as saying that the laws of everything (reality) are relative and sometimes seen as undermining the notion of absolute objective reality. But, as we shall discuss later, even relativity has absolutes.

General relativity and cosmology
We live in an evolving universe! Not only are space and time intertwined, the whole structure of the universe changes with time. The Universe had a beginning a finite time ago in the past. It might recollapse, thus lasting a finite time, or expand forever. See work by contemporaries Einstein, Hubble, and Lemaitre.

Quantum theory
The uncertainty principle (due to Heisenberg, not Einstein). Uncertainty in position/velocity, energy/time. Probabilistic nature of sub-atomic world. Cultural interpretation: all of reality is "fuzzy." See silly books like "The Tao of Physics." Einstein did *not* like the probabilistic

interpretation of quantum mechanics: "Herr Gott würfelt nicht!"

2 Special Relativity

First, let's review some history behind Special Relativity. Before SR, most physicists believed that there was an aether that filled space, upon which light waves traveled, just as sound waves or waves in the ocean need a substance to travel through [Draw some pictures of waves. What have you drawn?]. This would imply that the speed of light would look different if you moved at different velocity. If so, one could detect this variation! The presence of an aether throughout space would imply an absolute frame of reference, a special frame, with respect to which one could measure motion. Michelson and Morley found no such evidence. We'll discuss their experiment in Week 3.

Einstein's approach to Physics often involved the use of "thought experiments" (Gedanken-experiment), followed by the development of a mathematical theory to describe the Physics. His thoughts about space and time included the following insight: If there's no way to tell whether you're in a moving or stationary frame of reference, then there is no aether, no absolute notion of "at rest." One of Einstein's most famous thought experiments is the following: What would the speed of light look like if you traveled along on a photon? He postulated that the laws of physics should look the same to all freely-moving observers. This implies that the speed of a photon must be the same in every frame. "Freely-moving" simply means without any accelerations. From this he derived the laws of SR.

To be specific, Special Relativity can be deduced from just **two fundamental postulates:**

1. *Principle of Relativity*: No experiment can measure the absolute velocity of an observer.

2. *Universality of the speed of light*: The speed of light relative to any unaccelerated observer is the same, regardless of the observer's motion relative to the source of the light. In other words, all unaccelerated observers who observe a photon will measure c as its speed, regardless of their motion relative to one another.

The first postulate dates back to Galileo. The second, quite radical postulate was made by Einstein.

3 Implications and Applications

Some of the implications of SR include

- Clocks run at different rates for observers in relative motion (time dilation).
- Objects appear to have different sizes to observers in relative motion (length contraction).
- Matter and Energy can be transmuted into one another ($E = mc^2$).

In our explorations of the nature of time and space we'll encounter and fully discuss the so-called "Twin paradox." This describes time travel of a one-way sort: you can go to the future, but you can't come back.

Several "paradoxes" arise when we first examine the implications of SR. But there is no problem with SR – the paradoxes are cleared up as soon as we consistently apply SR. The apparent paradoxes are always due to faulty reasoning, usually involving inconsistent calculations.

Some practical applications of both Special and General Relativity include

- Clocks in airplanes
- orbiting satellites
- Space Shuttle experiments
- Navigation through Solar System

What's the difference between *Special* and *General* Relativity? Special Relativity describes physics as done/observed in frames (laboratories) moving at different constant velocity (unaccelerated). General Relativity "generalizes" SR to deal with accelerated frames and, therefore, gravity.

4 The Speed of Light

The understanding of mechanics that Isaac Newton described is not wrong, but rather it is now understood to be an approximation that is valid under the conditions usually observed. That is, Newtonian physics is nearly exact when the speeds of things are small. Before 1905, physicists thought of time and space as completely separate entities. What Einstein revealed is that time and space are intimately intertwined, although this connection is not obvious under the conditions that we normally observe in our daily lives.

When does SR become necessary? When we require exact calculations and when the velocities of objects and observers are significant when compared to the speed of light, $c = 2.99792458 \times 10^8$ m/s. This is exact because the meter is now defined so that c has this value. The second is defined by the period of oscillation of radiation from Cesium. 1s$=$ $9,192,631,770$T, where $T = 1/f$, f is the frequency of the hyperfine transition of the ground state of Cesium 133 (which must be cold!).

Approximation: The speed of light is almost exactly 3.0×10^8 m/s – close enough for this class. (Another fun fact that will help you to compute quickly: a year is almost exactly $\pi \times 10^7\,\mathrm{s}$. So, how many meters are in a light-year (the distance that light travels in one year)?).

We usually think of light as moving instantaneously from one place to another. To illustrate that the speed of light is finite consider some examples of light travel time. Recall $l = vt$, thus $t = l/v = d/c$.

- Across the room, 10 m, $t = 10\mathrm{m}/c = 3.33 \times 10^{-8}\,\mathrm{s} = 0.333$ ns
- To Sun, 8.32 minutes.
- To nearest star, 4 years.
- Across Milky way, 30 thousand years.
- To the nearest galaxy (Andromeda), million years.

My own research is in Astrophysics. The fact that light takes a finite amount of time to travel these distances becomes important, because it means that we see the universe not as it is, but as it was. For this reason, you are sitting at the oldest place in the universe that you can observe. A telescope is a time machine that allows you to look (but not travel) into the past!

When does SR become important? Not when distances between things are large but, rather, when velocities are significant. Specifically, we start to notice that Newtonian mechanics doesn't work when the ratio v/c is non-negligible. Some examples:

- Car at $60\mathrm{mph} = 27\mathrm{m/s}$, thus $v/c = 9.0 \times 10^{-8}$. No SR correction needed.
- Space shuttle or space station in orbit around Earth 1/90 minutes, $v = 1200$ m/s, thus $v/c = 4 \times 10^{-6}$. SR effects just start to become noticeable. Therefore clocks on the space shuttle run slow when compared to clocks on the ground (but there is also a general relativistic effect that speeds up the clocks - see next item. For the Space Shuttle, the special relativistic effect still dominates).

- GPS satellite: $v = 3.874\,\mathrm{km\,s^{-1}}$, $v/c = 1.3 \times 10^{-5}$. Also, altitude is $20,184\,\mathrm{km}$ (geosynchronous orbit), so there is a significant general relativistic effect. Combined special and general relativistic effects cause clocks on GPS satellites to run fast by 38 microseconds per day. In this case the clocks run fast because the GR effect that makes them run fast by 45 microseconds per day, is larger than the SR effect, which makes the moving clock run slow by 7 microseconds per day. Note: 1 microsecond$= 10^{-6}$s.

5 Distance Metric

First Key Concept: the invariance of distance between events in spacetime.
Definition: **spacetime** = three dimensions of space, plus one dimension of time, unified into one geometric framework
Definition: **invariant** = independent of observer

In Special Relativity we'll learn how to relate observations made by different observers and see that certain observed properties are independent of observer while others vary. If the observers are moving at constant velocity relative to each other, we'll see that the observations are related by making a coordinate transformation from one frame to another.

Sometimes the differences in observations are simply differences in the coordinates that are used. To illustrate this dependence, consider the parable of surveyors described in chapter 1 of the textbook. The two groups of surveyors in the town mark plots of land using different measuring systems. We'll see that these ways of measuring can be reconciled and that the *distance* between locations is independent of the coordinate system.

6 Surveyors Parable

Surveyors in this town use different units for measuring distances N-S or E-W. For reasons lost in the annals of time, it was traditional to measure North-South distances in miles and E-W in meters. Bizarre indeed!

Although they share use of the same measurement units, the two groups of surveyors, the Daytimers and Nightimers, use different methods for measuring where North is: magnetic North and the direction of the North star, respectively. The two groups reckon North differently by a small, but noticeable amount (1.15 degrees in this example). Because their coordinate systems are rotated, they obtain slightly different coordinates for the same locations on the ground. Because of these discrepancies, arguments ensued about boundaries between property, lawsuits about trees planted and buildings built.

For example – see Table 1-1, which lists the location of corner stakes as measured from the center of the town square. (DRAW picture and make TABLE of coordinates from page 3.) These coordinates differ by 40 to 60 meters - quite noticeable. Who was right?

The situation is confusing because of the mixtures of units and different coordinate systems. How do we make sense of measurements that differ in both magnitude and units? A clever student reconciled these by using correct conversion between units. He found that N-S and E-W coordinates varied, but distances were the same! In other words, the total distance is an *invariant* quantity. The total distance squared is the sum of the squares of the coordinate differences. This is just the Pythagorean theorem – DRAW the triangle,

$$d^2 = (k(n_2 - n_1))^2 + (e_2 - e_1)^2$$

where $k = 1609.344$ meters/mile

COMPUTE the distance from town center for stake C for both cases. You get the same distance regardless of coordinate system. This shows that the distance between points on the ground is invariant. The coordinates used may vary, but the distance between points on the ground is a real physical property of the land.

Notice again what was necessary to reconcile the two systems of measurement. We must take into account

- Rotation of coordinates
- Different units in different directions

Lecture 2: Thursday, January 10, 2018

0 Preliminaries

Required Reading: *Spacetime Physics*, ch. 1

Homework: HW1 is due on Thursday, January 17 at the beginning of class. Ch. 1, problems 4(a,b,c), 5, 8, 11, 12. See the web page for "homework hints." We'll do several examples today that will help.

Important Concepts:

- Recall Speed of Light and Surveyor's Parable from Lecture 1
- Unity of spacetime
- How to make correct observations
- Events and Spacetime Intervals
- Units

1 Review of the Surveyor's Parable

The surveyor's parable illustrates the physical conception of space and time before Einstein, Lorentz, Poincare. Special Relativity reveals the unity of "spacetime."

1. Different units for space, time. Spacetime is unified by converting time to distance, or vice versa using $c =$speed of light (in vacuum) to convert between meters and seconds: $l = ct$, thus $t = l/c$. This speed is a constant, independent of reference frame.
2. Different frames of references are related by a rotation of the coordinate system. In spacetime, these coordinate systems are tied to frames of reference that move at different velocity.
3. Invariance of the distance interval solves the discrepancy. Invariance of distance on the ground in this parable is analogous to invariance of intervals in space-time, as we shall see.

2 Surveying spacetime: John, Mary, and the Spacetime Interval

John stands in the doorway of a building while Mary rushes towards him in her spaceship. Look out John! As the spaceship passes John, a spark jumps from an antenna on the spaceship to the door frame. The spaceship flies down the hallway where another spark jumps from the antenna to a fire extinguisher on the wall, 2m from the doorway. What do John and Mary observe?

(DRAW THIS, work out math. Note the importance of using consistent, or at least sensible, numbers of significant digits in calculations.)

John observes two events located in his reference frame at (x_1, t_1) and (x_2, t_2). He measures a distance $x_2 - x_1 = 2.0000\,\mathrm{m}$ between the spark events – this is just the distance between door and fire extinguisher in his frame. Using precision clocks he measures a time interval $t_2 - t_1 = 33.6900 \times 10^{-9}\,\mathrm{s}$ between the spark events.

Mary, riding in the rocket, measures time $t'_2 - t'_1 = 33.0228 \times 10^{-9}\,\mathrm{s}$ between the spark events. In her frame the sparks both occur at the same location – the nose of her rocket – thus $x'_2 - x'_1 = 0$.

Before we go on, let's be careful about what we mean by "observe." For the purpose of this course, observations are always made locally, not as seen at a distance. When an event (for example, spark jumps from antenna to fire extinguisher) is observed, we mean that the location and time of that event are recorded at the location of the event. So, when the spark jump from the antenna to the fire extinguisher is observed, this is the same as saying that a video camera right next to the fire extinguisher captured the event on tape, with a time stamp on every frame of the tape. To find the exact time of the event, John examines the videotape. He does NOT compute time between events by using his eyeball and clicking a stopwatch on and off, because then the light travel time from the event to him would affect his measurement. We will return to this point later and discuss it in detail. For now, make sure that you understand that the observations of events are not affected by the time it takes light to travel from the event to a person.

Note details of what happens:

Both start their precision clocks when the antenna passes through the door frame. We'll use this event to set the *arbitrary* origin of their spacetime coordinate systems.

Both measure distance between the door spark and fire extinguisher spark in their own frame.

John (stationary) observes sparks separated in both space and time.

Mary (in rocket ship) observes sparks separated only in time.

The 2 events – sparks on door frame and fire extinguisher – have different spacetime coordinates in the two frames of reference.

How do we reconcile these different measurements?

Just as the distance interval and the correct conversion between units resolved the different coordinates measured by the Daytimers and Nightimers, so the use of the *Spacetime interval* (or Lorentz interval) and the correct conversion between time and space reconciles the event coordinates observed by John and Mary. The spacetime interval between events is defined to be

$$d^2 = (ct)^2 - l^2$$

where the speed of light c is expressed in ms^{-1}, distance l in m, and time t in s. Once you're used to measuring time and space in the same units, I'll stop writing the speed of light c in the equations, thus $d^2 = t^2 - l^2$. Note the minus sign in this definition for the spacetime interval. This equation is not the same as the Pythagorean theorem that you learned in geometry!

(DO MATH FOR JOHN vs. MARY and show that interval is same.)

The time interval in John's frame is $ct = (3.0 \times 10^8\,\mathrm{ms}^{-1})(33.69 \times 10^{-9}\,\mathrm{s}) = 10.1\,\mathrm{m}$. The space interval is $l = 2\,\mathrm{m}$. Thus $d^2 = (ct)^2 - l^2 = 102.01\,\mathrm{m}^2 - 4\,\mathrm{m}^2 = 98.01\,\mathrm{m}^2$, thus the spacetime interval is $d = 9.9\,\mathrm{m}$.

In Mary's frame, $ct = (3.0 \times 10^8\,\mathrm{ms}^{-1})(33.0228 \times 10^{-9}\,\mathrm{s}) = 9.9\,\mathrm{m}$. The space interval $l = 0$. Thus the square of the spacetime interval is $d^2 = (ct)^2 - l^2 = (9.9\,\mathrm{m})^2$ and, quite obviously, $d = 9.9\,\mathrm{m}$, matching the spacetime interval in John's frame.

What is the *relative* speed of John and Mary's reference frames? This is simply the speed of Mary's rocket, as observed by John. She moves $2\,\mathrm{m}$ in $33.69 \times 10^{-9}\,\mathrm{s}$, thus her speed in units of the speed of light is $v/c = (2\,\mathrm{m})/(33.69 \times 10^{-9}\,\mathrm{s})/(3.0 \times 10^8\,\mathrm{ms}^{-1}) = 0.198$, roughly two-tenths of the speed of light.

This strange way of measuring spacetime intervals (distance in spacetime) is the same regardless of reference frame. This is the invariant distance, which is the same for any observer. We'll derive this later in the course. Note that 3 space dimensions plus one time dimension = 4 dimensions! And note carefully the minus sign! Space and time are intertwined but are not completely interchangeable. They are still different physical entities.

Different frames correspond to different spacetime coordinate systems, rotated with respect to each other, but the rotation cannot be arbitrarily large, as we'll show later in the course.

Let's work another example:

Sparking at a Faster Rate example problem: A rocket flies down a hallway in which John is at rest. At each tick of the rocket's clock (which is not necessarily once per second!), a spark flashes from its antenna. Between a particular pair of flashes, John observes that the distance between the flashes is $x_2 - x_1 = 4.000\,\mathrm{m}$ and that the time between the flashes is $t_2 - t_1 = 16.6782048 \times 10^{-9}\,\mathrm{s}$.

(a) time in meters between events as measured by John?
$ct = (3.0 \times 10^8\,\mathrm{ms}^{-1})(16.6782048 \times 10^{-9}\,\mathrm{s}) = 5.000\,\mathrm{m}$

(b) spacetime interval measured by John?
$(ct)^2 - l^2 = (5.000\,\mathrm{m})^2 - (4.000\,\mathrm{m})^2 = 9.000\,\mathrm{m}^2$, thus $d = 3.000\,\mathrm{m}$.

(c) spacetime interval in rocket frame?
Even without knowing $t'_2 - t'_1$, we know that $d = 3.000\,\mathrm{m}$, by the invariance of d.

(d) distance between sparks in rocket frame?
$x'_2 - x'_1 = 0$, because the sparks occur at the same location – the rocket antenna.

(e) time in meters in rocket frame?
$(\text{spacetime interval})^2 = (\text{time interval})^2 - (\text{space interval})^2$. The space interval is zero, thus the time interval is identical to the spacetime interval, $c(t'_2 - t'_1) = 3.000\,\mathrm{m}$. In units of seconds, $t_2 - t_1 = 3.000\,\mathrm{m}/(3.0 \times 10^8\,\mathrm{ms}^{-1}) = 1.000 \times 10^{-8}\,\mathrm{s} = 10$ns.

(f) speed of rocket measured by John, in units v/c?
$v/c = x/(ct) = (4.000\,\mathrm{m})/(5.000\,\mathrm{m}) = 4/5$. In units of meters per second, $v = 2.4 \times 10^8\,\mathrm{ms}^{-1}$.

3 Events and Intervals

Events and intervals are real things and are all that matters for doing Physics. An event has a location and time as measured by each observer. The spacetime interval is an invariant of spacetime between events. Different observers might record different locations and times for events, but the spacetime intervals will be the same, regardless of observer! The coordinates given to an event by an observer don't mean very much by themselves; they have meaning only in relation to the coordinates of other events, since the zero points of both the spatial and time coordinates are arbitrary. Note carefully how we make observations:

We mark time with a clock that we hold in our hands or with a system of synchronized clocks that ride along in our frame.

We measure distance using a ruler that is tied to our frame of reference.

The time interval measured by a clock that is present at the location of two events is called the *proper time* or *local time*. In other words, proper time is the time between two events that occur at the location of the same clock (proper clock). For example, consider the wristwatch that you carry with you. As measured by a space and time coordinate system centered on you, all events that happen next to you are at the same same location, so the spacetime interval has no space contribution, hence the spacetime interval

$$d^2 = (ct)^2 - 0 = (ct)^2$$

In a *timelike* interval the spacetime interval is positive. There is more time than space between events.

Correspondingly, when there is more space than time (always in the same units!) then we say that the interval is *spacelike*. This seems to imply that the square of the spacetime interval is negative, because $l^2 > (ct)^2$.

For example, Using synchronized clocks, two people clap at the same time, one in New York, one in San Francisco. What is the spacetime interval?

$$(\text{spacetime interval})^2 = (3000 \text{ miles} \times 1609 \text{ m/mile})^2$$

but

$$(\text{time interval})^2 = 0$$

For events at extremely different locations and/or very short time separation we obviously can't have negative (distance)^2 so we define the square of the spacetime interval to be

$$d^2 = l^2 - (ct)^2 = l^2 - (0)^2 = l^2$$

for this case (or we could simply take the absolute value in all cases).

4 Units

To emphasize the unity of spacetime in this course, we'll often use time measured in meters! Sounds strange, eh? Earlier we discussed distances measured in units of light travel time:

Across the room, 10 m, $t = 10\text{m}/c = 3.33 \times 10^{-8}$, 0.333 ns

To Sun, 8.32 light-minutes.

To nearest star, 4 light-years.

Across Milky way, 30 thousand light-years.

To the nearest galaxy (Andromeda), million light-years.

These units make sense for describing large distances because the large value of c makes them manageable. In regular distance units the distance to the nearest galaxy is $\sim 3.0 \times 10^{21}$ meters.

For objects that move at velocities close to the speed of light, using meters of time will likewise prove useful, since then the tiny fractions of a second, like the nanosecond times in the examples above, are translated into numbers that are easier to work with. And then we don't need to keep multiplying, dividing by c. We're not just making this up for teaching this class. Physicists often convert to such units when working in Special and General Relativity. In fact, so-called "Geometrodynamical" units use $c = 1$, $G = 1$.

5 Sample Problems

Now let's work a couple of the "Proton, Rock, and Starship" examples.

Sample problem 1-2a (p. 14): A proton at $v = 3/4c$ goes through two detectors, 2.000m apart. As the proton passes through each detector, a light flashes on the detector. Detector one sits at x_1, detector two sits at x_2, where $x_2 - x_1 = 2.000$m.

What are the space and time separations between the flashes in the lab frame?

In the lab frame the two events occur at (x_1, t_1) and (x_2, t_2). The space interval is simply $x = x_2 - x_1 = 2$m. In time t, an object at velocity v travels $x = vt$, thus $t = x/v$. Thus the time interval is simply $t_2 - t_1 = 2.000\text{m}/(3/4c) = 2.6667\text{m}$ of time.

What are the space and time separations between the flashes in the proton frame? Label the proton frame spacetime coordinates of the events (x'_1, t'_1) and (x'_2, t'_2).

In the proton frame, which moves along with the proton, both flashes occur at the same location, thus the space interval $x'_2 - x'_1 = 0$.

To find the time interval between the events in the proton frame, we use the invariance of the spacetime interval. We require $d^2 = d'^2$, or

$$(\text{lab time interval})^2 - (\text{lab space interval})^2 = (\text{proton time interval})^2 - (\text{proton space interval})^2$$

Now plug in the intervals from above, remembering that the proton space interval is zero. Thus,

$$(\text{proton time interval})^2 = (2.667\,\text{m})^2 - (2.000\,\text{m})^2 = 3.1111\,\text{m}^2$$

and the proton time interval $t'_2 - t'_1 = 1.7638\,\text{m}$ of time. Note that this is *shorter* than the time interval in the lab frame.

Sample problem 1-2c: A rocket leaves Earth (event 1) at $v = 0.95c$ and arrives at Proxima Centauri (event 2), 4.3 light-years away.

What are the space and time separations in years as measured in the Earth frame?

The space interval is simply $x_2 - x_1 = 4.3\,\mathrm{ly}$. Using $x = vt$, the time interval is $t_2 - t_1 = 4.3\,\mathrm{ly}/0.95\,\mathrm{ly}/\,\mathrm{y} = 4.53\,\mathrm{y}$.

What are the space and time separation in the rocket frame?

Both launch and arrival occur at the same place in the rocket frame, thus $x'_2 - x'_1 = 0$. To find the time interval in the rocket frame, use the invariance of the spacetime interval:

$$(\text{rocket time})^2 - (\text{rocket distance})^2 = (\text{Earth time})^2 - (\text{Earth distance})^2$$

Rocket distance is zero and we found the Earth time and distance above, thus

$$(\text{rocket time})^2 = (4.53\,\mathrm{y})^2 - (4.3\,\mathrm{y})^2 = 2.03\,\mathrm{y}^2$$

so rocket time is $t'_2 - t'_1 = 1.42\,\mathrm{y}$.

Note that a round trip at 0.95c returns the space travelers to Earth a full 6.22 years younger than Earth-bound colleagues born at the same time. Doesn't seem fair – get to see the universe AND stay young!

The "unity of spacetime": $d^2 = t^2 - l^2$ for time, space in same units. See quote from Minkowski (p. 15) This is perhaps a slight exaggeration – space and time are not identical e.g., note the minus sign in the interval.

Problem 1-7 (p. 22): Spacetime Map from the Exercises.

Again, solve this using only the invariance of the spacetime interval. (What are "spacetime intervals?")

A Simple Example: A rocket travels at constant velocity. It flashes a light when it passes by point 1 in the lab frame and again at point 2. In the lab frame, the distance between points 1 and 2 is $x_2 - x_1 = 3\,\mathrm{m}$. The lab time between these events is observed to be $t_2 - t_1 = 5\,\mathrm{m}$ of time. What are the space and time intervals between the flashes as observed in the rocket frame? Quite obviously, $x'_2 - x'_1 = 0$ because the light on the rocket is always at the same location relative to the rocket pilot. The invariance of the spacetime interval implies that the rocket pilot observes a time interval between the events

$$(\text{rocket time})^2 = (5\,\mathrm{m})^2 - (3\,\mathrm{y})^2 = 16\,\mathrm{m}^2$$

so rocket time is $t'_2 - t'_1 = 4\,\mathrm{m}$.