PHYSICS 233: INTRODUCTION TO RELATIVITY
Winter 2018-2019
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Homework assignment 8 Solutions

Chapter 7: 7-2, 7-8, Chapter 8: 8-8
Extra credit: 8-40

7-2 System Mass See figure 7-6 on p. 207.
Total mass of system depends on total energy and total momentum, $M^2 = E^2 - p^2$. Total energy $E = 13 + 17 = 30$. Total momentum $p = -5 + 15 = 10$. Thus system mass $M = \sqrt{30^2 - 10^2} = \sqrt{800} = 28.28$.
Sum of individual masses is $\sum m_i = 8 + 12 = 20$. Not the same as system mass.
Mass of system before and after interaction is the same, $M = 28.28$. This must be true since $M^2 = E^2 - p^2$ and both E and p are conserved. For the same reason, the momenergy 4-vector also does not change.

7-8 Rocket nucleus See figure 7-8 on p. 216. $m_A = 20, m_C = 2, E_C = 5$.

(a) $E^2 = m^2 - p^2$, $p = 0$, thus $E_A = m_A = 20$.

(b) Energy before collision equals energy after, $E_A = E_C + E_D$, $E_D = E_A - E_C = 20 - 5 = 15$.

(c) $m^2 = E^2 - p^2$, thus $p_C^2 = E_C^2 - m^2$. C moves to left, $p_C = -\sqrt{5^2 - 2^2} = -4.58$.

(d) Momentum before collision equals momentum after, all particles move along same axis, $0 = p_C + p_D$, $p_D = -p_C = 4.58$.

(e) $m_D = \sqrt{E_D^2 - p_D^2} = \sqrt{(15)^2 - (4.58)^2} = 14.28$.

(f) $m_C + m_D = 2 + 14.28 = 16.28$, which is smaller than $m_A = 20$. The residual mass of the system lies in the internal kinetic energy of the C and D system.

(g)

8-8 Nuclear Excitation

(a) The nucleus has $m = 1.00$, $E = m$, $p = 0$. The photon has $m = 0$, $E_\gamma = p_\gamma$. Thus the total energy is $E_{sys} = E_\gamma + 1$ and the system momentum is $p_{sys} = E_\gamma$. The mass of the system is therefore $m_{sys} = \sqrt{(E_\gamma + 1)^2 - E_\gamma^2}$. After absorbing the photon, the nucleus has rest mass $m = 1.01$. In the frame of the nucleus, this is the same as the system mass, which is invariant, thus $1.01 = \sqrt{2E_\gamma + 1}$, which we solve to find that $E_\gamma = 0.01005$, just slightly larger than the increase in rest mass.

(b) The photon energy must be larger than the increase in rest mass because, in order to conserve momentum, the photon's energy must be split between increasing the rest mass and the kinetic energy of the nucleus (it has to leave moving off to the right in the diagram).

Extra credit: 8-40 Power paddlewheel on cart

First of all, let's assume that this system is built with the same sort of frictionless pullies and other parts that we learned about in high school. Also no wind resistance. And assume that the puddle of water is perfectly insulated, so that no heat can get out.

(a) If the motor does work at a rate dE/dt, then the rate of mass transfer to the paddlewheel end is simply dE/dt.

(b) At low velocities, $p = mv = mdx/dt$ We're moving mass $dm = dE$ over distance x in time dt, thus the momentum is $p = x(dE/dt)$.

(c) Mass transfer is from motor to paddle, thus momentum of this mass transfer is in the same direction, to the right. Conservation of momentum requires that the board roll to the left. When the battery runs down, the momentum to the right stops. By conservation of momentum, the board must also stop. Think about this as a system: it began with zero momentum, it must end with zero momentum.

How far will the board move in this time? It must have moved just enough so that the center of mass of this system has not changed.

(d)

Observer on board sees only the belt move, thus energy must be transferred by belt.

Observer on table sees both belt and board moving, thus he infers that energy is transferred by belt and board.

Observer one way on belt sees other side of belt moving and the board moving, so he sees that energy is transferred by belt moving in other direction and by board.