PHYSICS 233: INTRODUCTION TO RELATIVITY
Winter 2018-2019
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Homework Assignment 7 Solutions

Chapter 6: 6-1, 6-3, 6-4

6-1 Relations between Events

(a) Events 1 and 2:

(1) Proper time $\tau^2 = t^2 - x^2 = 5^2 - 3^2$, thus $\tau = 4$.

(2) Yes, event 1 could cause event 2 because the required velocity between the events is $v = 3/5 < 1$.

(3) Yes. This requires $x'_2 - x'_1 < 0$. Look at the Lorentz transformation for these two events: $x'_2 = \gamma(x_2 - v_{rel}t_2) = \gamma(3 - 5v_{rel})$, $x'_1 = \gamma(x_1 - v_{rel}t_1) = \gamma(1 - 2v_{rel})$, thus $x_2 - x_1 = \gamma(2 - 3v_{rel})$. If $v_{rel} > 2/3$, then events are spatially reversed

(4) No. Again, look at the Lorentz transformation: $t'_2 = \gamma(-v_{rel}x_2 + t_2) = \gamma(-3v_{rel} + 5)$ and $t'_1 = \gamma(-v_{rel}x_1 + t_1) = \gamma(-v_{rel} + 2)$, thus $t_2 - t_1 = \gamma(-2v_{rel} + 3)$. This can't be negative unless $v_{rel} > 3/2$, so no frame achieves $t_2 < t_1$.

Events 1 and 3:

(1) These events have a spacelike interval, so quote the proper distance $s^2 = x^2 - t^2 = 5^2 - 3^2 = 4^2$, thus $s = 4$.

(2) No. A signal would have to travel at $v = 5/3 > 1$.

(3) No. Using similar math as in (3) above, this requires $x'_3 - x'_1 = \gamma(5 - 3v_{rel}) < 0$, which can't happen unless $v_{rel} > 5/3$.

(4) Yes. As above, we require $t'_3 - t'_1 = \gamma(-5v_{rel} + 3) < 0$, which occurs for $v_{rel} > 3/5$.

Events 2 and 3:

(1) Spacetime interval is $t^2 - x^2 = 0$.

(2) Yes, a signal at light speed from 3 could reach 2.

(3) No. That would require $x'_3 - x'_2 = \gamma(2 + 2v_{rel}) < 0$.

(4) No. That would require $t'_2 - t'_3 = \gamma(2v_{rel} + 2) < 0$.

(b) Using the Lorentz transformtion as above, events 1 and 2 occur at the same place in a frame with $v_{rel} = 3/5$. Events 1 and 3 occur at the same time in a frame with $v_{rel} = 3/5$.

6-3 Proper Time and Proper Distance

(a) If P and Q have a spacelike separation, then $x > t$ between them. Use the Lorentz transformation as in 6-1 above: $t'_P - t'_Q = \gamma(-(x_P - x_Q)v_{rel} + (t_P - t_Q)) = 0$ when $v_{rel} = (t_P - t_Q)/(x_P - x_Q)$. Since $x_P - x_Q > t_P - t_Q$ for a spacelike interval, there is always some such frame. In this frame, the proper distance between the events is simply $s^2 = x'^2 - t'^2 = x'^2$.

(b) For a timelike separation, the events occur at the same place if $x'_P - x'_Q = \gamma(-(t_P - t_Q)v_{rel} + (x_P - x_Q)) = 0$, thus $v_{rel} = (x_P - x_Q)/(t_P - t_Q)$ which is less than one for all timelike intervals. In this frame, the proper time between the events is $\tau^2 = t'^2 - x'^2 = t'^2$.

6-4 Autobiography of a Photon

Follow the discussion suggested in the book and consider the flight of a proton (not a photon) that travels the $10^5\,\mathrm{ly}$ at different velocities v. In the galaxy frame, the distance is $x = 10^5\,\mathrm{ly}$ and the trip takes time $t = 10^5\,\mathrm{ly}/v\,\mathrm{y}$. In the frame of the proton, the distance is $x' = x/\gamma = 10^5\,\mathrm{ly}\sqrt{1-v^2}$ and the trip takes time $t' = t/\gamma = 10^5\sqrt{1-v^2}/v\,\mathrm{y}$. As the energy of the proton increases, v increases. As v approaches one, the distance between the stars approaches zero in the proton frame. Likewise, the time between emission and absorption approach zero in the proton frame. We infer that, in the photon frame, the emission and absorption events occur at the same place and time.

(a) My autobiography is rather brief. When I'm travelling in a vacuum, my birth and death occur in the same place at the same time. And you thought *your* life was boring.

(b) In the astronomer frame, the photon travels $x = 10\mathrm{km} = 10^4\,\mathrm{m}$ at velocity $v = 1/1.00030$, which takes time $t = 1.00030 \times 10^4\,\mathrm{m}$. In the photon frame, it spends time $t' = t/\gamma = \sqrt{1-(1/1.00030)^2}(10^4)(1.00030) = 245\,\mathrm{m}$. This time is the proper time, as measured by the photon. The distance travelled is $x' = x/\gamma = \sqrt{1-(1/1.00030)^2}10^4 = 245\,\mathrm{m}$.

(c) Wow, something finally happened to me! I zipped through 245 meters of atmosphere in 245 meters of time and ended my brief existence being absorbed by a molecule in the eye of an astronomer. What a way to go.