PHYS 233: INTRODUCTION TO RELATIVITY
Winter 2018-2019
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Homework Assignment 6 Solutions

Spacetime ch 5 problems 5-2, 5-4, 5-6.

5-2 Transforming Worldlines

(a) First the easy part: In the lab frame, worldline B is of a particle that doesn't move, so in the rocket frame, which moves in the negative x direction relative to the lab, this particle appears to move in the positive x direction, so B's worldline must tilt to the right. In the rocket frame, the intercept of B's worldline with the x' axis will be closer to $x' = 0$ than in the lab frame, due to length contraction.

Now for the tricky part. Note that the point of tracing the hyperbolae is that the transformed events 1, 2, and 3 must lie on these curves in the spacetime diagram of any other frame. Event 0 lies at the same place in both frames, since it's at $x = 0$, $t = 0$. Since the rocket moves in the negative x direction, the velocity from 0 to 1 must appear larger in the rocket frame, thus event 1 must lie further up and to the right along its invariant hyperbola. Event 2 occurs at the same place as event 1 in the rocket frame, and so must lie on its invariant hyperbola directly above event 1. The same argument for event 1 applies to event 3: the velocity from 2 to 3 must appear larger in the rocket frame, so event 3 lies up and to the right on it's hyperbola so that the segment 2 to 3 has a larger angle with respect to the time axis than in the lab frame diagram.

Also see the attached sketch.

(b) In the rocket frame, the particle has zero velocity between 1 and 2, which means that this new frame must have velocity $v_{rel} = (x_2 - x_1)/(t_2 - t_1) = (1.75 - 3.00)/(7.0 - 4.0) = -1.25/3 = -0.417$. In the lab frame, worldline B is of a particle that doesn't move, so in the rocket frame, this particle appears to move in the positive x direction at $v = 0.417$.

(c) The spacetime interval between events should be the same in both frames, so we can use the lab frame coordinates to compute the intervals. The "proper times" labelled on the lab frame diagram are equivalent to the total interval in any other frame, so just copy these onto your rocket frame diagram.

Further notes: You can check your intuition regarding where the events lie in the rocket frame by applying the Lorentz transformation to a pair of events, say, 0 and 1. Again, event 0 stays at $x' = 0$ and $t' = 0$. Event 1 lands at $x' = \gamma(x - v_{rel}t) = (1 - (0.417)^2)^{-1/2}(3.0 - (-0.417)4.0) = 5.14$ and $t' = \gamma(-v_{rel}x + t) = (1 - (0.417)^2)^{-1/2}(-(-0.417)3.0 + 4.0) = 5.78$. Indeed, in the rocket frame, event 1 lies at larger space and time separation from event 0. The velocity of the particle along segment 0 to 1 is $v' = x'/t' = 0.89$, compared to $v = x/t = 0.75$.

5-4 Pole and Barn Paradox

A 20 meter pole inside a 10 meter barn? In the barn frame, the pole shrinks to 10 meters long and so just barely fits inside the barn, thus the farmer can close both doors at once with the pole inside. But from the pole frame, the barn shrinks to only 5 meters. No way it fits inside, right? As you suspect, the key to resolving this "paradox" is in the relativity of simultaneity. In the farmer's frame, both doors close at the same time. But in the pole and runner's frame, the doors are never closed at the same time. In the pole's frame, the front door closes *after* the leading end of the pole goes out the back door.

(a) Let's use spacetime diagrams to examine this situation more closely.

The relative velocity of the barn and pole must satisfy $\gamma = (1 - v_{rel})^{-1/2} = 2$, thus $v_{rel} = 0.866$.

In the barn frame, both the barn and pole are 10 meters long. The worldline of the front door (location A in the diagram on p. 166) lies long the time axis $x = 0$. The wordline of the back door (location B), lies along the line $x = 10\,\mathrm{m}$. The worldline of the pole's leading edge, Q, lies along a line that includes $x = 0$, $t = 0$, where it enters the front door, and $x = 10\,\mathrm{m}$, $t = 10/0.866 = 11.55\,\mathrm{m}$, when it reaches the back door, B. Thus, in the barn frame, Q reaches B after time $t = 11.55\,\mathrm{m}$. The worldline for the pole's trailing edge, P, extends from $x = -10\,\mathrm{m}$, $t = 0$ (where is must be when Q is at A), to $x = 0$, $t = 11.55\,\mathrm{m}$. Thus, P coincides with A at time $t = 11.55\,\mathrm{m}$ and P coincides with B at time $t = 23.1\,\mathrm{m}$. In the barn frame, "Q at B" and "P at A" both occur at time $t = 11.55\,\mathrm{m}$, so the pole is inside the barn at this instant and the farmer can slam the front door shut, with the pole inside. And let's be kind and assume that the back door is made of paper, so that the runner slices through without injury.

In the pole frame, the pole is 20 meters long and the barn is only 5 meters long. Remember that both barn and pole frames use "Q is at A" as the zero of their spacetime coordinates. The leading end of the pole, Q, follow a worldline that lies on the time axis, $x' = 0$. The worldline of the trailing end of the pole, P, is the line $x' = -20\,\mathrm{m}$. The front door, A, moves on a worldline that passes through $x' = 0$, $t' = 0$ but is tilted so that is moves at velocity $v = -0.866$. The back door, B, also has velocity $v = -0.866$, but passes through $x' = 5\,\mathrm{m}$, $t' = 0$. Thus, in the pole frame, Q coincides with B at time $t' = 5/0.866 = 5.77\,\mathrm{m}$. Also, P and A coincide at time $t' = 20/0.866 = 23.1\,\mathrm{m}$ and P and B coincide at time $t' = 28.87\,\mathrm{m}$. Now, examine when "Q at B" and "P at A" occur in the pole frame. "Q at B" is at time $t' = 5.77\,\mathrm{m}$. But "P at A" occurs at $t' = 23.1\,\mathrm{m}$. If the farmer shuts the door when P reaches A, this occurs long after the leading edge reached the back door. In other words, the leading end, Q, bursts through the paper back door long before the farmer shuts the front door behind trailing end P.

(b) What if there's no back door, just a concrete wall? In the barn frame, the pole smashes into the wall the instant after the front door closes. In the pole frame, the pole also smashes into the wall. But it does so *before* the tailing edge P ever enters the barn. Does the pole get into the barn or not? The event "Q at B" occurs, then a shock is sent along the pole toward P telling it "hey, the pole hit a wall!" Can that signal get to P before P gets to A? Look at the pole's spacetime diagram. "Q at B" and "P at A" are separated by $x' = 20\,\mathrm{m}$ of space (the lenght of the pole) and $t' = 23.1 - 5.77 = 17.3\,\mathrm{m}$ of time. The interval is spacelike. So, the signal would have to travel at $v' = 20/17.3 > 1$, faster than the speed of light. The pole DOES get into the barn, albeit in lots of pieces.

(c) To make this clear, replace the pole with a sequence of tennis balls, spread over 10 meters in the barn frame, spread over 20 meters in the pole frame. In the barn frame, the events of the children catching the ten tennis balls occur along the line connecting "P at A" with "Q at B" with the first ball at B and the last at A. These events are simultaneous in the barn frame.

In the pole frame, the events of the children catching the balls are also spread along a line connecting the events "P at A" with "Q at B" with the first ball caught at B and the last at A. The interval between any pair of the stopping events is spacelike, thus the information that the first ball has been stopped does not reach the second ball until after it, too, is stopped. Likewise, the last ball does not learn that any of the other balls has been stopped until *after* it has stopped. After the last ball has been stopped, it sees that the other balls have been caught by the children.

5-6 A Summer's Eve Fantasy

More fun with spacetime diagrams!

If the aliens travel such that $\gamma = 5/3$, then their velocity must be $v = 4/5 = 0.8$.

The outbound alien ship travel with $\gamma = 25/7$, thus $v = 24/25$.

All units are minutes or light-minutes.

(a) Plot these events:

0. At the origin

1. You see this at $x = 0$, $t = 0$ and the light from Sun, which lies at $x = -8$, travels at $v = 1$, so it must have occurred at $t = -8$.

2. Light from Sun explosion reaches you at same place and time as aliens landing, $x = 0$, $t = 0$.

3. Distance from Sun to Venus is $8-2 = 6$ light-minutes. The blast wave travels at $v = 0.5$, so Venus's atmosphere is blown away $6/0.5 = 12$ minutes after Sun explodes. This occurs at $x = -2$, $t = -8 + 12 = 4$.

4. Light from event 3 reaches us at $v = 1$. Venus is 2 light-minutes away, so we receive the signal at $x = 0$ and $t = 4 + 2 = 6$ minutes.

5. We depart from $x = 0$ at $t = 7$.

6. Blast wave from Sun must travel 8 light-minutes at $v = 0.5$, taking 16 minutes, and reaches Earth ($x = 0$) at $t = -8 + 16 = 8$ minutes. So, we DO get away in time.

(b) Worldlines

A. Follows time axis at $x = 0$ up to event 5 at $v = 0$, then follows a line at $v = 24/25$.

B. Follows time axis at $x = 0$ $v = 0$.

C. Line with $v = -4/5$ that ends at event 0.

D. Parallel to time axis at $x = -8$, $v = 0$. Sun explodes, so worldline ends at $t = -8$.

E. Parallel to time axis at $x = -2$, $v = 0$.

F. Line at $v = 1$ from event 1 to 0 (also in opp. direction).

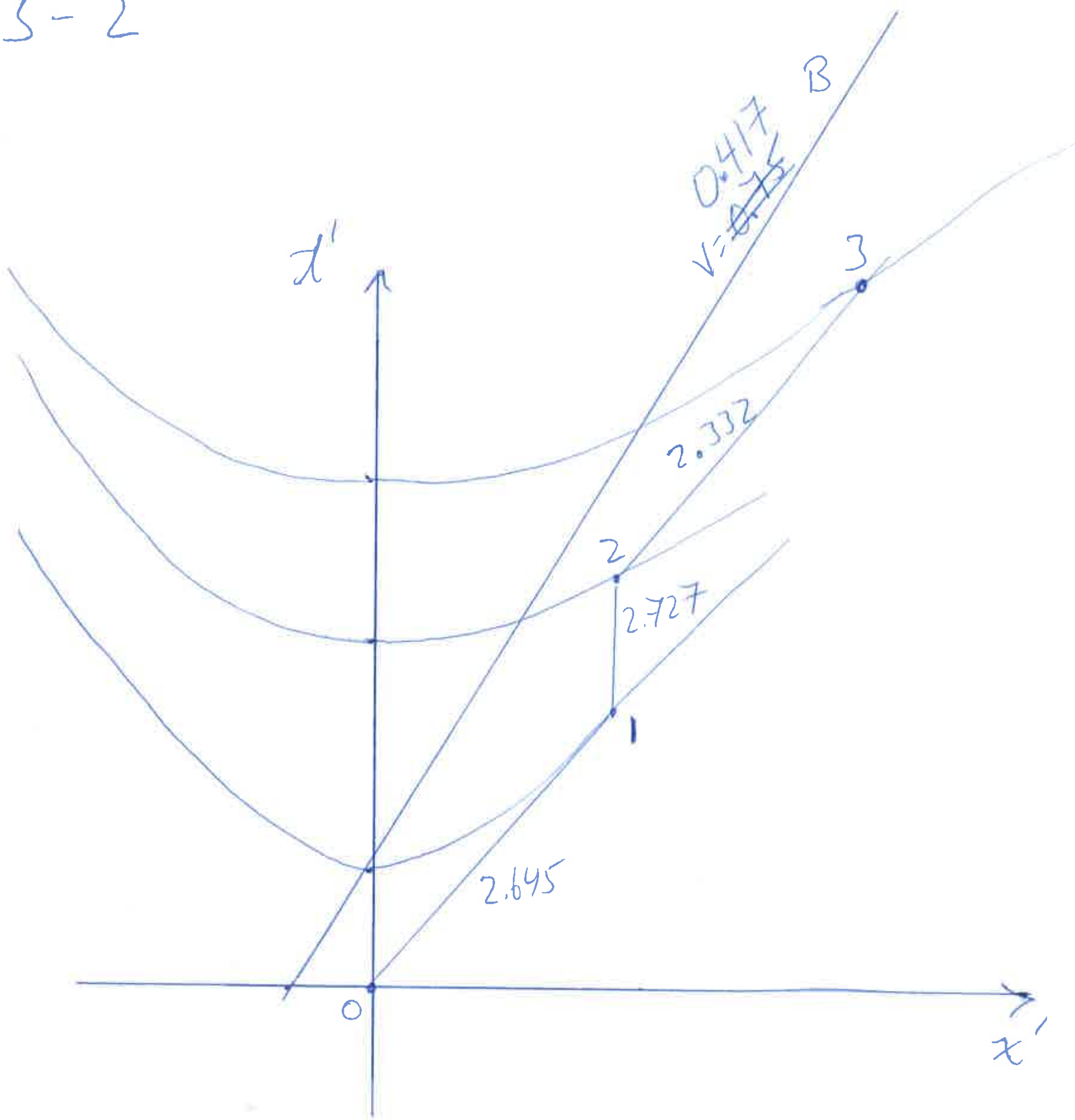
G. Line at $v = 0.5$ from 1 to 3 and on to 6 (also in opp. direction).

H. Line at $v = 1$ from 3 through 4 (also in opp. direction).

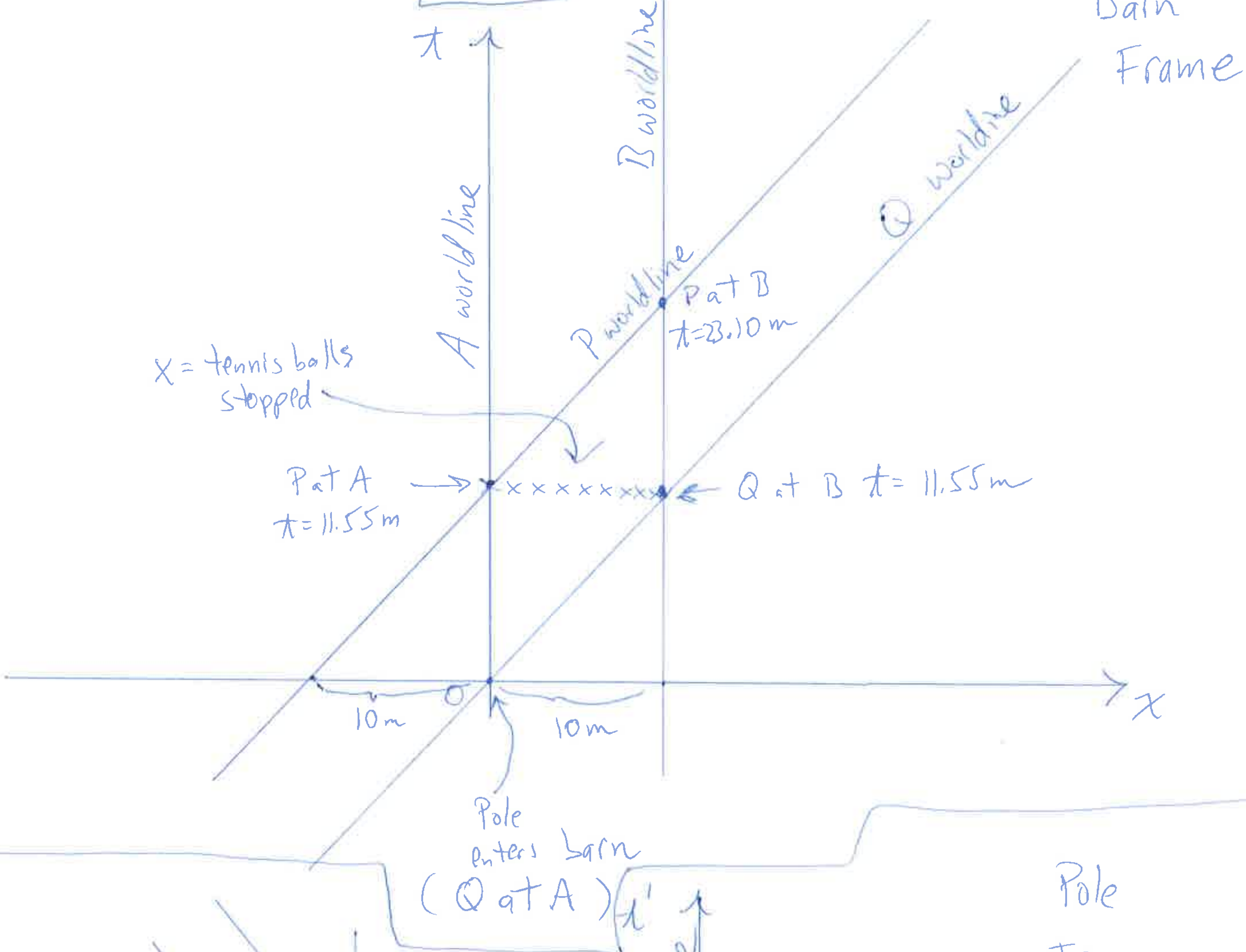
J. Reaches event 1 at $v = -1$, connects down to line C at lower right.

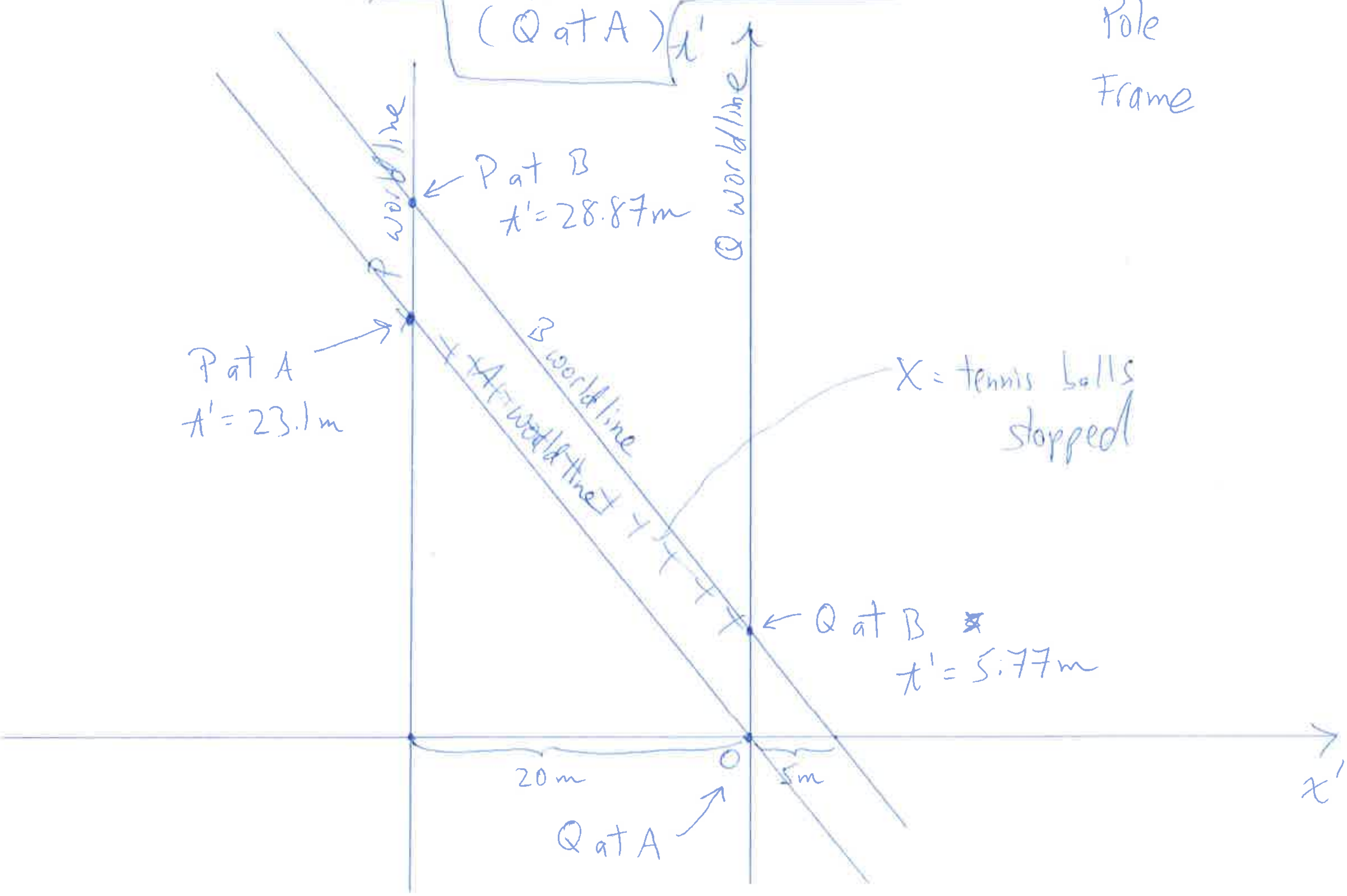
See attached sketch.

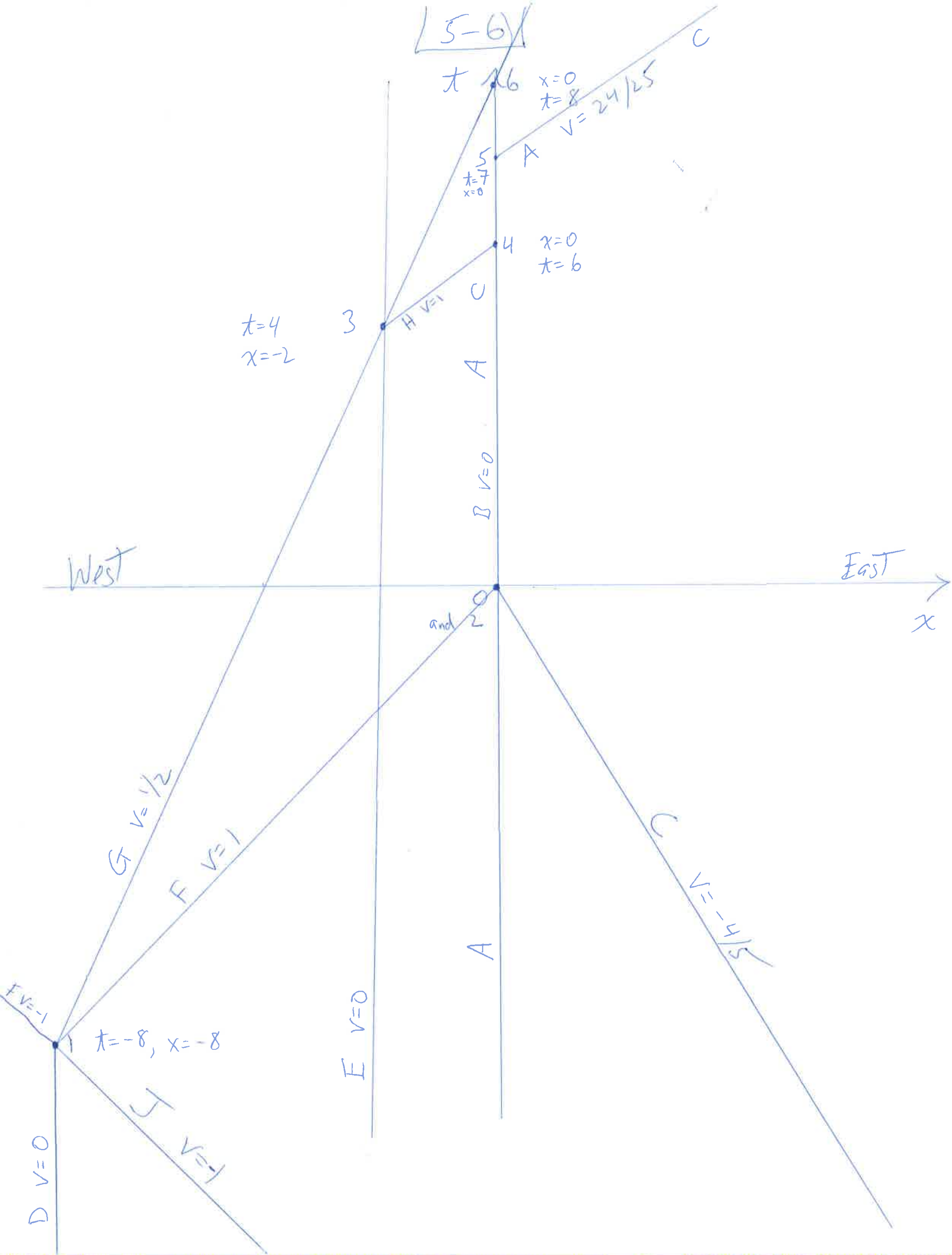
5-2



5-4


Barn Frame
t
x
A worldline
B worldline
P worldline
Q worldline
P at B
t=23.10 m
x = tennis balls stopped
P at A
t = 11.55 m
Q at B t = 11.55 m
10 m
10 m
O
Pole enters barn
(Q at A)


Pole Frame
t'
x'
P worldline
Q worldline
A worldline
B worldline
P at B
t' = 28.87 m
P at A
t' = 23.1 m
X = tennis balls stopped
Q at B
t' = 5.77 m
20 m
5 m
O
Q at A


5-6
t
6
x=0
t=8
V=24/25
C
A
5
t=7
x=0
4
x=0
t=6
C
H V=1
3
t=4
x=-2
A
B V=0
West
East
x
0
and 2
G V=1/2
F V=1
C
V=-4/5
A
F V=-1
1
t=-8, x=-8
E V=0
C
V=-1
D V=0