INTRODUCTION TO RELATIVITY
Winter 2018-2019
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Homework Assignment 5 Solutions

Spacetime ch. 4, problem 1.

4-1 Practical Space Travel Remember to set the zero for all clocks at the moment James leaves Earth.

In the Earth frame:

a) James reaches Sirius at time $t = x/v = 8.7\,\mathrm{ly}/0.75 = 11.6\,\mathrm{y}$.

b) James stays at Sirius for 7 years as measured on his clock. No time dilation for this, because he's sitting still in the Earth-Sirius reference frame while he waits to return. Thus, he leaves for Earth at $t = 11.6\,\mathrm{y} + 7\,\mathrm{y} = 18.6\,\mathrm{y}$.

c) James arrives back at Earth at $t = 18.6\,\mathrm{y} + 11.6\,\mathrm{y} = 30.2\,\mathrm{y}$.

In James' reference frame:

d) James arrives at Sirius after $t' = t/\gamma = 11.6\,\mathrm{y}/(1/\sqrt{1-(0.75)^2}) = 7.67\,\mathrm{y}$.

e) He leaves Sirius after 7 years, at $t' = 7.67\,\mathrm{y} + 7\,\mathrm{y} = 14.67\,\mathrm{y}$. Remember that he and his clock are in the Earth frame while hanging around Sirius.

f) James arrives back on Earth at $t' = 14.67\,\mathrm{y} + 7.67\,\mathrm{y} = 22.34\,\mathrm{y}$.

g) The lookout stations in James' moving rocket frame measure a contracted length for the Earth-Sirius separation, $x' = x/\gamma = 8.7\,\mathrm{ly}/(1/\sqrt{1-(0.75)^2}) = 5.76\,\mathrm{ly}$.

h) When James reaches Sirius, all clocks that are synchronized in his frame read the same time, $t' = 7.67\,\mathrm{y}$. Simultaneous with his arrival at Sirius the rocket-frame clock Q is next to Earth and observes a clock on Earth to read $t = 7.67\,\mathrm{y}/\gamma = 5.07\,\mathrm{y}$.

Not sure about this last result? Sounds like we've done a "double time dilation?" Don't worry, but be careful. In the Earth frame, the event of James reaching Sirius occurs at time $t = 11.6\,\mathrm{y}$ because that's how long it takes a ship at 0.75c to go 8.7 light-years. In James' frame, the distance is length-contracted along the direction of travel, so from his point of view he sees a "rod" of contracted length 5.76 light-years fly by at 0.75c in time 7.67 years. Now the tricky part: Using camera-clocks synchronized in James' frame, he uses the camera-clock nearest Earth to observe an Earth clock to read 5.07 years when he reaches Sirius. Why doesn't it read 11.6 years? Because, although James reaching Sirius and observing the Earth clock are simultaneous in his frame, those events are NOT simultaneous in the Earth frame. This observation of the Earth clock appears at a different time and place than the Earth frame observation, made with an Earth-frame synchronized camera clock near Sirius, of the event of James arriving at Sirius.

i) On his return journey, James travels in an incoming frame that has clocks synchronized to his that are spread along the direction of travel. Remember that this is a different inertial frame (incoming rather than outgoing velocity!) than the one he used to get to Sirius. When James leaves Sirius to return home, camera-clock Z is next to Earth. At that instant, clock Z reads the same as his clock, $t' = 14.67\,\mathrm{y}$. Using camera-clock Z to peer in on Earth, it observes an Earth clock to read $t = 30.2 - 5.07 = 25.1\,\mathrm{y}$.

Confused again? Why does the Earth clock read 25.1 years when James leaves Sirius? We computed above that the trip takes 30.2 years as measured in the Earth frame. And we showed that, as observed by camera-clocks that are synchronized in James' frame, an Earth clock ticks off 5.07 years in the time for James to travel from Earth to Sirius, or vice versa. Therefore, the Earth clock must read $30.2 - 5.07 = 25.1$ at the beginning of his journey home, as observed by a rocket-frame synchronized camera-clock.