PHYSICS 233: INTRODUCTION TO RELATIVITY
Winter 2018-2019
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Homework Assignment 3 Solutions

Spacetime ch. 3, problems 3-2, 3-4, 3-12(a, b only) 3-14 (a, b only), 3-17
Due Thursday, January 31.

2. Runner in the Mirror Will the runner see herself in the mirror? Yes! Ignore her moving legs and think of her and the mirror sitting in an inertial frame that moves with respect to the ground. Regardless of her speed with respect to the nearby ground, light still moves at the speed of light between her and the mirror and there is no problem (apart from not paying attention to where she's going) with observing her own reflection.

4. The Principle of Relativity Yes means "same in both frames," No means "could be different in two frames."

(a) Yes. The speed of light in a vacuum is constant in all frames.

(b) Yes. The spacetime interval $t^2 - x^2$ is an invariant and therefore identical in both free-fall frames.

(c) No. Kinetic energy depends on velocity, which is not the same in all frames. Think of the frame of the electron itself, in which the electron has zero velocity and thus zero kinetic energy.

(d) Yes. The rest mass of the electron is a constant of nature and measures the same in all frames. Otherwise we could measure our velocity by measuring variation of the electron mass, in violation of the Principle of Relativity.

(e) No. The magnetic field strength measured at a point in space looks different to different frames. Think of measuring this field by its effect: measure the force, which involves measuring acceleration, which is a change in velocity. Velocities appear different depending on one's frame, thus the force appears different and one infers a different strength for the field.

(f) No. The distance between events can vary if there is a component of this distance that lies along the direction of relative motion between the two frames.

(g) Yes. Structure of DNA, the arrangement of molecules that determine the genetic structure of a living organism, does not change with the velocity of the frame. Do you turn into a duck at high velocity? Thus, the functional structure of DNA does not change. But does it *look* different to an observe in another frame? Yes, there is length contraction, so the answer to this question could also be "No" if you explained it this way.

(h) No. The rate of change of momentum observed depends on measurements of the velocity of the neutron.

12. Airplane Travel in Strong Wind ("Michelson-Morley Experiment")

Suppose that light travelled like sound waves, so that c is the speed relative to the medium in which the light travels. If you were moving relative to that medium, then the apparent speed of light would vary and you could tell if you were moving with respect to that medium.

Michelson and Morley designed an experiment to detect the Earth's motion with respect to this medium, called the "aether." The following problem illustrates how this phenomenon affects air travel, because planes make progress by flying through the air; their groundspeed is affected by the motion of the air.

(a) The travel time in one direction is $t = x_{AB}/v_{net}$ where v_{net} is the net velocity with respect to the ground. With no wind, $v_{net} = c$, thus t_{trip} is just twice the one-way time, thus $t_{trip} = 2x_{AB}/c$.
Now add the wind: Flying against the wind from A to B, $v_{net} = c-v$, thus $t_{AB} = x_{AB}/(c-v)$. Flying with the wind on the way back, $t_{BA} = x_{AB}/(c+v)$. The total time of flight is then

$$t_{trip} = \frac{x_{AB}}{c+v} + \frac{x_{AB}}{c-v}$$

Simplify this equation by factoring out c in the denominators of the terms on the right, then adding the terms together (review high school algebra), thus

$$t_{trip} = \frac{2x_{AB}}{c}\frac{1}{1-v^2/c^2}$$

Comparing to the no-wind time, we see that the wind causes the trip time to incease by a factor $1/(1-v^2/c^2)$.
The round trip time is longer because, although the wind increases the return speed by the same amount as it slowed the plane on the flight out, the plane spends more $time$ in the air at the slower speed, so the average velocity of the roundtrip is not the same as the average of the velocities on the two legs.
When $v \approx c$, the plane takes off and hovers above the ground near its point of departure, never actually arriving at B. This limiting case makes it clear that the problem is that flying against the wind takes more time, even if the return speed is faster. In the continental United States, airlines overcome this problem a bit by flying in the jet stream from West to East, but avoiding it when flying from East to West, if possible.

(b) On the trip from A to C, the plane must overcome a crosswind of v, which means that its velocity must have a component that is perpendicular to its intended direction of travel. In other words, the plane has to fly with its nose pointed diagonally. Its total velocity is still c. Use the Pythagorean theorem to solve for the velocity along the intended direction of travel:

$$v_{AC} = \sqrt{c^2 - v^2}$$

Thus, the travel time $t_{AC} = x_{AC}/\sqrt{c^2-v^2}$. The same applies on the return trip, thus $t_{trip} = 2x_{AC}/\sqrt{c^2-v^2}$. Factor out c from within the square root to find that

$$t_{trip} = \frac{2x_{AC}}{c}\frac{1}{\sqrt{1-v^2/c^2}}$$

With no wind, the trip time would be simply $t_{trip} = 2x_{AC}/c$, so the trip takes a factor of $1/\sqrt{1-v^2/c^2}$ longer when there is a crosswind.

14. Moving Faster than the Speed of Light?

(a) Scissors Paradox: When the rod moves in the y direction a distance $\Delta y = v_{rod}\Delta t$, the point of intersection with the x axis moves a distance $\Delta x = \Delta y/\tan\theta$. Thus, the velocity of the intersection of the rod with the x axis is $v_A = v_{rod}/\tan\theta$. For small values of θ, $\tan\theta$ approaches zero, so the velocity v_A can be arbitrarily large, even greater than the speed of light when $v_A = v_{rod}/\tan\theta > c$. But no object, no energy, thus no information moves as the point A changes position, so you can't use this to send signals at $v > c$.

(b) Hammer Pulse: No. As in (a), once the whole rod is in motion, the point of intersection with the x axis can appear to move at $v > c$. However, no information travels at this speed. Why? Consider how you set the rod in motion: The impact of the hammer sends a signal along the rod, but this pulse within the rod moves at the speed of sound along the rod, not at the superluminal velocity of the point A.

JUST FOR FUN...here are answers to parts c and d:

(c) Searchlight Messenger: Let's put the observers A and B both at distance d_{search} from the searchlight. Draw a line from the searchlight to each observer. Call the angle between the lines θ. Using trigonometry we find that the distance from A to B is $d_{AB} = 2d_{search}\sin(\theta/2)$, over which light travels in $t_{AB} = d_{AB}/c$. If the searchlight rotates at ω radians per second, then the searchlight will sweep from A to B in time $t_{search} = \theta/\omega$. Thus, for $t_{search} < t_{AB}$, we obtain $\theta/\omega < 2d_{search}\sin(\theta/2)/c$ and the distance from the observers to the searchlight must be $d_{search} > c\theta/(\omega 2\sin(\theta/2))$.
No, the warning message has not travelled faster than the speed of light. Think of the searchlight hitting A and then B as two signals, first to fire the bullet, then to duck. To send the "fire" signal from the searchlight, the searchlight is turned a bit, and some photons travel straight toward A, reaching A after time d_{search}/c. To send the "duck" signal, the searchlight keeps turning, and the searchlight lights up B when photons released at the moment when the searchlight was pointed at B have travelled d_{search}/c. Thus, although A and B receive their instructions – "fire" and "duck" – only a fraction of a second apart, both signals travelled from the searchlight to their recipients at the speed of light, no faster.
Look at it another way: Can A use the searchlight to send a signal to B at faster than light speed? No. To tell the searchlight to point at B requires sending a signal to the searchlight, then for the photons at the searchlight to reach B. If A holds his own searchlight and waves it at B, the photons from his light also travel at light speed.

(d) Oscilloscope: Yes, the bright spot can move across the oscilloscope screen at $v > c$ for the same reason that the searchlight can sweep from A to B faster than a light signal between A and B in part (c) above. Rememember, each spot on the oscilloscope screen is caused by different photons hitting the screen. No individual photon moves across the screen.

17. Contraction or Rotation? The answers are in the back of the book, so pay attention to getting these answers, not just copying from the book! Yes, this is a hard problem.

(a) For light from points G and E to reach O at the same time, the light from E must have left 1 meter of time earlier, since it's 1 meter of distance further away from O (no length contraction in this direction, perpendicular to the direction of motion). Thus, the cube moves a distance $x = v(1\,\mathrm{m})$ for time measured in meters, where x is also the distance in the figure.

(b) Now the weirdness begins. Since the light from E left before G, the observer sees the trailing side of the cube at the same time that he sees the bottom (note carefully our language here: "sees" literally means what a person sees, not what synchronized rods and clocks would measure, as we usually try to do). The observer sees a piece of the trailing side that has width x in projection (see the figure on p. 93). The bottom side of the cube appears length contracted by the factor $\sqrt{1-v^2}$. This is how a stationary cube would look if it were rotated! Looking at the figure, it is clear that the apparent rotation angle ϕ is such that $x = (1\,\mathrm{m})\sin\phi$, where $x = v(1\,\mathrm{m})$ from (a). Therefore $v = \sin\phi$. Check that for $v = 0$, the apparent angle $\phi = 0$. For motion at the speed of light, $v = 1$, the apparent rotation is a full 90 degrees – we would see the trailing side of the cube rather than its bottom as it passed overhead.

(c) Discussion: What does "really" mean? The observations of each observer are perfectly valid. A lattice of rods and clocks in the rocket frame does not detect any rotation – the edges of the cube remain aligned with the rods. An observer in the rocket frame also does not "see" with this eyes any rotation. In the lab frame, properly synchronized observerations measure length contraction of the cube along the direction of travel, but no rotation. In the lab frame, the observer sort of "sees" rotation because of the lack of simultaneity of his observations of the near and far corners of the cube. Is it "really" rotated? The lab observer does see the back side of the cube. That phenomenon is certainly "real" in the sense that this is what a person with one eye closed would see.

(d) We cheated in (b) and (c) and assumed that the lab observer has no depth perception. Including depth perception, the observer can tell that points G and H are the same distance from him, rather than at different distances as implied by the lower right figure on p. 93. Likewise, points E and F are not shifted in distance. Thus, an observer with proper depth perception sees the distance side EF of the cube pulled towards the trailing edge, the whole cube sheared like a parallelogram rather than rotated.