# PHYSICS 233: INTRODUCTION TO RELATIVITY
## Winter 2018-2019
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## Homework Assignment 2 Solutions

Spacetime Physics chapter 2, problems 4, 9, 10, 11, 13.

NOTE: In the word problems, you *must* justify your answers. Simply answering "yes" or "no" is not sufficient. You must always explain why, just as you must always provide calculations to back up your numerical answers. The answers to the odd-numbered problems are in the book; copying those answers is not a very good way to learn the material.

### 4. Synchronization by traveling clock?

**(a)** Can Mr. Englesburg synchronize his clocks any way he wants? Of course not. Incorrect methods for "synchronization" will cause him to make observations that are not easily reconciled with those of observers in other inertial frames. He can check his method of synchronization by a simple observation: He looks at each clock through a telescope and compares the time on the clock to the time on his wristwatch. The time on the distant clock should be behind (earlier than) his wristwatch by exactly the light travel time from him. For example, a clock that is 100m away had better appear slow by $t = 100\,\mathrm{m}/(3\times10^8\,\mathrm{ms}^{-1}) = 3.33\times10^{-7}\,\mathrm{s}$.

**(b)** No, Big and Little Ben are not synchronized. When the moving clock reaches Little Ben, the elapsed time on the moving clock will be different from the elapsed time on Big Ben, thus Little Ben will be incorrectly set. Why? From the frame of Big and Little Ben, the moving clock runs slow (remember "time dilation" means that the length of time between clock ticks always seems longer in frames in which the clock is *not* at rest).

**(c)** If the moving clock flies at $v = 10^5\,\mathrm{ms}^{-1}$, then it takes $t = 10^9\,\mathrm{m}/(10^5\,\mathrm{ms}^{-1}) = 10^4\,\mathrm{s}$ to travel between Big and Little Ben, as measured in the frame of the Bens. The spacetime interval between the two events, moving clock passes Big and moving clock passes Little Ben, is (working in time and distance units of seconds here) $t^2 - x^2 = (10^4\,\mathrm{s})^2 - (10^9\,\mathrm{m}/(3\times10^8\,\mathrm{ms}^{-1})^2 = 10^8\,\mathrm{s}^2 - 11\,\mathrm{s}^2$. This must equal the spacetime interval in the moving clock frame, in which both events occur at the same place, $t'^2 - x'^2 = t'^2 - (0)^2 = (10^8 - 11)\,\mathrm{s}^2$. Do the algebra and find that $t' - t = -5.5\times10^{-4}\,\mathrm{s} = -0.55\times10^{-3}\,\mathrm{s}$. When the moving clock passes Little Ben and sets it, it will be running 0.55 millisecond slow relative to Big Ben.

**(d)** At 100 times faster, $v = 10^7\,\mathrm{ms}^{-1}$, the travel time in the Bens' frame is $t = 10^9\,\mathrm{m}/(10^7\,\mathrm{ms}^{-1}) = 10^2\,\mathrm{s}$. Again using the spacetime interval, the time elapsed in the moving clock frame is $t'^2 = t^2 - x^2 = (10^2\,\mathrm{s})^2 - 11\,\mathrm{s}^2$. Again, do the algebra and find that $t' - t = -5.5\times10^{-2}$, 55.0 milliseconds slow. This is starting to get noticeable.

**(e)** Does it matter? That depends on how accurately the clocks must be synchronized. Given a specification for the accuracy of the synchronization, we can always move the moving clock slowly enough to achieve the desired accuracy (carry the clock on a turtle's back...).

### 9. Rising Railway Coach

**(a)** The ball bearings will move toward each other, decreasing their separation, because there is a sideways relative acceleration from the Earth.

**(b)** With one ball bearing above the other, they will move apart because the gravitational acceleration on the bearing nearer the Earth is larger than on the higher (further from Earth) bearing, thus there is a relative acceleration between them.

**(c)** No. The magnitude of the two effects described above, the bearings drawing closer together in the first case, and apart in the second case, depend only one the distance of the train from the center of the Earth, not on whether the train is falling up or down. For the same reason, you will not notice any change when the train stops rising and starts falling.

### 10. Test Particle?

**(a)** The acceleration on the smaller mass is $a = F/m = (6.67\times10^{-11}\,\mathrm{m}^3\,\mathrm{s}^{-2}\,\mathrm{kg}^{-1})(10\,\mathrm{kg})/(0.1\,\mathrm{m})^2 = 6.67\times10^{-8}\,\mathrm{m\,s}^{-2}$. At constant acceleration $a$ for time $t$, an object moves a distance $x = at^2/2$. Thus, in 3 minutes the smaller mass moves $x = (6.67\times10^{-8}\,\mathrm{m\,s}^{-2})(180\,\mathrm{s})^2/2 = 1.1\times10^{-3}\,\mathrm{m}$. Thus, it moves by 1 mm in just under 3 minutes.

**(b)** Ball bearings vary in size and thus in weight. I'll do this for a bearing that weighs 10 grams, or $m = 0.01\,\mathrm{kg}$. They begin a distance $r = 20\,\mathrm{m}$ apart. The gravitational acceleration of one bearing on another is $a = F/m = (6.67\times10^{-11}\,\mathrm{m}^3\,\mathrm{s}^{-2}\,\mathrm{kg}^{-1})(0.01\,\mathrm{kg})/(20\,\mathrm{m})^2 = 1.7\times10^{-15}\,\mathrm{m\,s}^{-2}$. In 8 seconds, it will move $x = at^2/2 = (1.7\times10^{-15}\,\mathrm{m\,s}^{-2})(8\,\mathrm{s})^2/2 = 5.4\times10^{-14}\,\mathrm{m}$. Not likely that we'd notice this.

### 11. Communications Storm

Through his telescope (e.g., via an optical flash from the Sun that travels at light speed), the astronomer sees evidence for an incoming burst of charged particles that will disrupt communications. It takes 3 minutes, or 180 seconds, to switch to the secure underground system on Earth. It takes 500 seconds for the optical flash to reach Earth. For the burst of particles to reach Earth with enough time to prepare, the particles can take no less than $t = 500\,\mathrm{s} + 180\,\mathrm{s}$ to arrive. They travel 500 light-seconds in at least 680 seconds of time, thus they travel at speed no greater than $v = 500/680 = 0.735$c

### 13. Deflection of Starlight

**(a)** A photon of light feels the Sun's gravity for an effective time that is roughly the diameter of the Sun divided by the speed of light, thus $t_{eff} \approx 1.4\times10^9\,\mathrm{m}/(3.0\times10^8\,\mathrm{ms}^{-1}) = 4.67\,\mathrm{s}$. If the light just grazes the Sun's surface, then the acceleration felt by a photon is $a = GM_{Sun}/(d_{Sun}/2)^2 = (6.67\times10^{-11}\,\mathrm{m}^3\,\mathrm{s}^{-2}\,\mathrm{kg}^{-1})(2\times10^{30}\,\mathrm{kg})/(0.7\times10^9\,\mathrm{m})^2 = 272\,\mathrm{m\,s}^{-2}$. The net velocity of the "fall" of the photon is then $v = at = (272\,\mathrm{m\,s}^{-2})(4.67\,\mathrm{s}) = 1271\,\mathrm{ms}^{-1}$.

**(b)** The photon moves at $v_y = 1271\,\mathrm{ms}^{-1}$ and $v_x = 3\times10^8\,\mathrm{ms}^{-1}$. For small angles, as measured in radians, $\theta \approx v_y/v_x = 4.2\times10^{-6}$, which is $2.4\times10^{-4}$ degrees.