PHYSICS 233: INTRODUCTION TO RELATIVITY
Winter 2018-2019
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Homework assignment 1 Solutions

Spacetime ch.1, problems 4a,b,c, 5, 8, 11, 12 (on pages 21-24)

4. Rocket emits two flashes of light

Here we use the invariance of the spacetime interval $d^2 = (ct)^2 - l^2$, to compute the missing quantity. This invariance of spacetime intervals implies $(ct_{rocket})^2 - (l_{rocket})^2 = (ct_{lab})^2 - (l_{lab})^2$. Here the distances are given in light-seconds, so we'll use distance measured in light-seconds and time measured simply in seconds, thus $(t_{rocket})^2 - (l_{rocket})^2 = (t_{lab})^2 - (l_{lab})^2$. The flashes of light are always at the same place in the rocket frame, so $l_{rocket} = 0$. In each case, write down the equivalence of the spacetime intervals and rearrange the equation to solve for the missing value.

(a) $(t_{rocket})^2 = (t_{lab})^2 - (l_{lab})^2 = (10.72\,\mathrm{s})^2 - (5.95\,\mathrm{s})^2 = 79.52\,\mathrm{s}^2$, thus $t_{rocket} = 8.92\,\mathrm{s}$.

(b) $(t_{lab})^2 = (t_{rocket})^2 + (l_{lab})^2 = (20.00\,\mathrm{s})^2 + (99.00\,\mathrm{s})^2 = 10201\,\mathrm{s}^2$, thus $t_{lab} = 101.00\,\mathrm{s}$.

(c) $(l_{lab})^2 = (t_{lab})^2 - (t_{rocket})^2 = (72.90\,\mathrm{s})^2 - (66.80\,\mathrm{s})^2 = 852.17\,\mathrm{s}^2$, thus $l_{lab} = 29.19\,\mathrm{s}$.

5. Two firecrackers in the lab

Again, we use the invariant spacetime interval to infer the observations in different frames of reference, $(t_{rocket})^2 - (l_{rocket})^2 = (t_{lab})^2 - (l_{lab})^2$. This time the two events occur at the same place in the lab, $l_{lab} = 0$, but at different places in the rocket frame. Just to keep you paying attention, here we work in time units of years – so it's easiest in this case to work with distance in light years. Remember, the speed of light in these units is just $c = 1$ light-year per year.

(a) $(l_{rocket})^2 = (t_{rocket})^2 - (t_{lab})^2 = (5\,\mathrm{y})^2 - (3\,\mathrm{y})^2 = (16\,\mathrm{y})^2$, thus $l_{rocket} = 4\,\mathrm{y}$

(b) In the rocket frame, the site of the exploding firecrackers travels $4\,\mathrm{y}$ of space in $5\,\mathrm{y}$ of time, so the relative velocity of the frames is $4/5$ the speed of light.

8. Light-speed limits on computing

One limit on the speed of a computer is the rate at which information moves around inside it. Information can never travel faster than the speed of light!

(a) One megaflop is 10^6 floating point operations per second, or one calculation every $10^{-6}\,\mathrm{s}$. Let's assume that, for each such calculation, the information must travel from the RAM to CPU and back. If it takes exactly $10^{-6}\,\mathrm{s}$ to make the trip, then the longest possible round trip, assuming that the signal propagates at the speed of light in a vacuum, is $2l = ct = (3.0 \times 10^8\,\mathrm{m s^{-1}})(10^{-6}\,\mathrm{s}) = 300\,\mathrm{m}$, so the maximum distance is $l = 150\,\mathrm{m}$. If the signal travels through the computer at merely half the speed of light, then the distance must be half as long.

(b) One gigaflop is 10^9 calculations per second, or one every $10^{-9}\,\mathrm{s}$. Redo the calculation from (a) and we find that the distance must be 10^3 times shorter, or $l = 0.15\,\mathrm{m}$, or 15 cm

(c) One teraflop is 10^{21} calculations per second, or one every $10^{-12}\,\mathrm{s}$. Redo the calculation from (a) and we find that the distance must be 10^6 times shorter, or $l = 1.5 \times 10^{-4}\,\mathrm{m}$, merely 0.15 mm.

(d) Suppose that our computational task is "trivially" parallelizable, so that we simply divide the number of calculations among a large number of equivalent processors. If the calculation time within each CPU is significantly longer than the time to move data and instructions from the "boss" process to the "slave" processors and back, then indeed we can achieve a gain in speed that is nearly proportional to the number of slave processors. But, if the calculation requires significant "interprocess" communication between the boss and slaves and/or among the slaves, then the speed of parallel computing can be dominated by the information travel time within the computer. The efficiency of dividing the task among processors can be significantly smaller, depending on the ratio of interprocess communication time to CPU calculation time. Another issue for parallel computing is the geometry of the arrangement of processors. If we pack the processors together as tightly as possible, in a roughly spherical assembly with the boss processor at the center, then the average interprocessor distance is proportional to the inverse cube of the density of processors $l \propto n^{-1/3}$, where n is the number of processors per m^3. However, it's hard to keep such a computer properly cooled (and hot computers run slower), so one must resort to lower-dimensional arrangements, which spread the processors further apart. Note that the famous Cray supercomputers have the shape of a torus.

11. Fast-moving particles last longer, μ mesons

In one half-life of a particle, half of a group of such particles decay. Imagine a burst of μ mesons, called muons, is created in the upper atmosphere and that a pack of them move together at the same speed directly downward, their numbers steadily declining as they near the Earth's surface. In one half-life, the number of muons decreases by a factor $1/2$. After N half-lives, the remaining number is proportional to $(1/2)^N$. Only because of the shorter travel time as seen in the muon frame do any of these particles reach sea level.

(a) If their velocity is very close to the speed of light, then the muons reach the Earth in $t = (60 \times 10^3\,\mathrm{m})/3.0 \times 10^8\,\mathrm{ms}^{-1} = 2.0 \times 10^{-4}\,\mathrm{s}$.

(b) In the muon's rest frame (traveling with them on their descent), their half-life is $t_{half} = 1.5 \times 10^{-6}\,\mathrm{s}$. If this half-life were the same for the Earth observers, then they would count $N = (2.0\times 10^{-4}\,\mathrm{s})/(1.5\times 10^{-6}\,\mathrm{s}) = 133$ half-lives. In that case a mere $(1/2)^{133} = 9.18\times 10^{-41}$ of the muons would reach other. In other words, none of them.

(c) If 1/8 of the muons reach sea level, then only 3 half-lives have elapsed, $1/8 = (1/2)^3$.

(d) In the frame of the muon, which is falling freely toward the Earth, its creation in the atmosphere and impact on Earth occur in the same place.

(e) From (c), three half-lives elapsed in the muon's frame, so it's time interval is simply $t_{muon} = 3(1.5 \times 10^{-6}\,\mathrm{s}) = 4.5 \times 10^{-6}\,\mathrm{s}$. The space interval is zero, so the spacetime interval is the same as the time interval.

12. Fast-moving particles last longer, π^+ mesons

These particles decay much faster than muons, so you have to observe them quickly!

(a) If there were no time dilation due to SR, then a pack of pions moving at the speed of light could travel a maximum of $l = ct_{half} = (3.0 \times 10^8\,\mathrm{ms}^{-1})(18 \times 10^{-9}\,\mathrm{s}) = 5.4\,\mathrm{m}$ before their number were halved.

(b) Use the invariance of spacetime intervals and some algebra. Start with $(ct_{pion})^2 - (l_{pion})^2 = (ct_{lab})^2 - (l_{lab})$. The events in question are the creation of the pions and their arrival at a point where they have traveled after one half-life in the pion frame. We know that $l_{pion} = 0$, since the pions don't move in their own frame. We also know that the lab observer sees the pions travel $l_{lab} = 0.9978ct_{lab}$ in time t_{lab}. Thus, for two events separated by t_{half} and $l_{pion} = 0$ in the pion frame, $(ct_{half})^2 = (ct_{lab})^2 - (0.9978ct_{lab})^2$. Divide out the factors of c and rearrange as $t_{half}^2 = t_{lab}^2(1-(0.9978)^2)$. Thus, $t_{lab} = t_{half}/\sqrt{1-(0.9978)^2} = 15.08t_{half}$. The time elapsed in the lab frame is 15.08 times longer than in the pion frame, therefore the distance traveled is 15.08 times as long.