

20.RQ.29

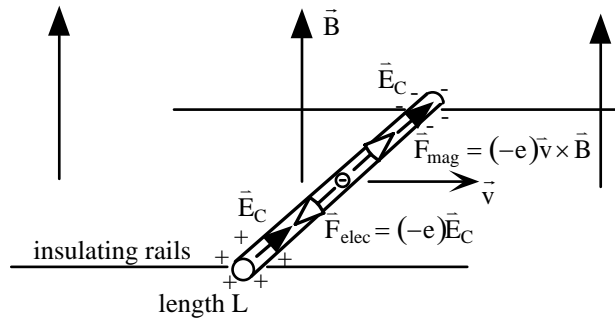


For the force to be inward (to turn the positively charged nucleus into a circular orbit), the magnetic field must be out of the page. For the magnitude, with  $r$  = radius of orbit and  $T$  = period (time for one revolution):

$$m \frac{v^2}{r} = F = 2evB \sin(90^\circ) = 2evB \quad (\text{nonrelativistic approximation})$$

$$B = \frac{mv}{2er} = \frac{m \left( \frac{2\pi r}{T} \right)}{2er} = \frac{\pi n}{eT} = \frac{\pi \left( \frac{4 \times 10^{-3} \text{ kg/mole}}{6 \times 10^{23} \text{ nuclei/mole}} \right)}{(1.6 \times 10^{-19} \text{ C})(80 \times 10^{-9} \text{ s})} = 1.6 \text{ tesla}$$

20.RQ.36



Magnetic force  $(-e)\vec{v} \times \vec{B}$  on an electron inside the rod polarizes the rod as shown.

A Coulomb electric field  $\vec{E}_C$  appears inside the rod as shown, due to the surface charges.

Polarization proceeds until  $F_{\text{electric}} = F_{\text{magnetic}}$  :

$$eE_C = evB \sin(90^\circ) \Rightarrow E_C = vB$$

$\vec{E}_C$  is uniform throughout the rod, since  $\vec{v}$  and  $\vec{B}$  are the same everywhere. The surface charges will arrange themselves to make a uniform electric field.

$$\Delta V \equiv -\int \vec{E}_C \cdot d\vec{l} = E_C L = vBL, \text{ since } \vec{E}_C \text{ is uniform.}$$

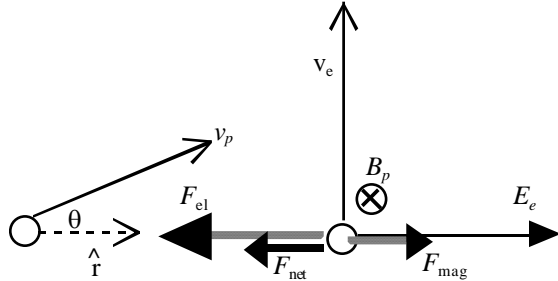
$$\Delta V = emf - r_{\text{internal}} I = emf, \text{ since } I = 0, \text{ so } emf = E_C L = vBL \text{ (1 pt).}$$

$$\text{Or, } emf = \int \frac{\vec{F}_{NC} \cdot d\vec{l}}{q} = \frac{evBL}{e} = vBL = E_C L.$$

Once the rod is polarized,  $I = 0$ , so there is no  $ILB$  magnetic force opposing the motion. So no force is required to keep the rod moving at a constant speed  $v$  on the frictionless rails.

**Problem 20.P.44 Moving proton and moving electron**

(a)



Force on electron:

Proton makes  $B$  into page at location of electron.

$$B_p = \left| \frac{\mu_0}{4\pi} \frac{e\vec{v}_p \times \hat{r}}{r^2} \right| = \frac{\mu_0}{4\pi} \frac{ev_p \sin \theta}{r^2} \text{ into page}$$

Magnetic force on electron by proton is to right.

$$F_{\text{mag}} = |-e\vec{v}_e \times \vec{B}_p| = ev_e B_p \sin 90^\circ = \frac{\mu_0}{4\pi} \frac{e^2 v_p v_e \sin \theta}{r^2}$$

Electric force on electron by proton:

$$F_{\text{el}} = |-e\vec{E}_p| = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2} \text{ to left}$$

Net Force:

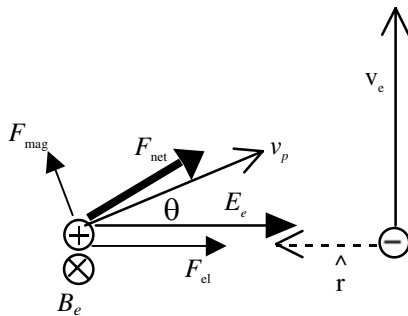
$$\vec{F}_{\text{net}} = \vec{F}_{\text{el}} + \vec{F}_{\text{mag}}$$

$$F_x = -\frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2} + \frac{\mu_0}{4\pi} \frac{e^2 v_p v_e \sin \theta}{r^2}$$

$$= -\frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2} \left[ 1 - \frac{v_e v_p \sin \theta}{c^2} \right] \text{ (since } \mu_0 \epsilon_0 = \frac{1}{c^2} \text{)}$$

$$F_y = 0$$

(b)



Force on proton:

Electron makes  $B$  into page at location of proton.

$$B_e = \left| \frac{\mu_0}{4\pi} \frac{-e\vec{v}_e \times \hat{r}}{r^2} \right| = \frac{\mu_0}{4\pi} \frac{ev_e}{r^2} \text{ into page}$$

Magnetic force on proton by electron is to northwest:

$$F_{\text{mag}} = |e\vec{v}_p \times \vec{B}_e| = ev_p B_e \sin 90^\circ = \frac{\mu_0}{4\pi} \frac{e^2 v_p v_e}{r^2}$$

Electric force on proton by electron:

$$F_{\text{el}} = |e\vec{E}_e| = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2} \text{ to right}$$

Net Force:

$$\vec{F}_{\text{net}} = \vec{F}_{\text{el}} + \vec{F}_{\text{mag}}$$

$$F_x = +\frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2} - \left( \frac{\mu_0}{4\pi} \frac{e^2 v_p v_e}{r^2} \right) \sin \theta$$

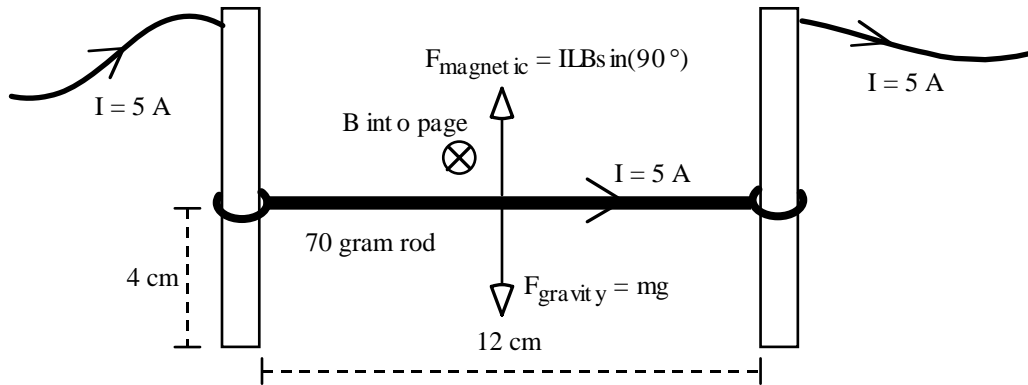
$$= \frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2} \left[ 1 - \frac{v_e v_p \sin \theta}{c^2} \right] \text{ (since } \mu_0 \epsilon_0 = \frac{1}{c^2} \text{)}$$

$$F_y = \left( \frac{\mu_0}{4\pi} \frac{e^2 v_p v_e}{r^2} \right) \cos \theta$$

(c) It is clear from the diagrams above that the net force on the proton is NOT equal and opposite to the net force on the electron! This is confirmed by the calculation; the  $x$  components of the force are equal and opposite, but the  $y$  components are not. Evidently Newton's third law (reciprocity) is not obeyed by magnetic forces.

(d) Total momentum of the particles is NOT conserved; they experience DIFFERENT momentum changes.

**Problem 20.P.52 Magnetic levitation**



In order to balance the downward force of gravity, there must be an upward magnetic force of equal magnitude, which requires a magnetic field into the page (right hand rule, with conventional current to the right).

$$ILB \sin(90^\circ) = ILB = mg$$

$$B = \frac{mg}{IL} = \frac{(0.07\text{kg})(9.8\text{N/kg})}{(5\text{ampere})(0.12\text{m})} = 1.14 \text{ tesla}$$

There could be an additional component of magnetic field to the right or left, which would exert no force on the wire:  $|\vec{IL} \times \vec{B}| = ILB \sin(0^\circ) = 0$ .

$$E = \frac{0.73 \text{ volts}}{0.15 \text{ m}} = 4.9 \text{ volts/meter}$$

$$u = \frac{v}{E} = \frac{4.8 \times 10^{-3} \text{ m/s}}{\left( \frac{0.73 \text{ volts}}{0.15 \text{ m}} \right)} = 9.9 \times 10^{-4} \frac{\text{m/s}}{\text{volts/m}}$$

Note that the electric field  $E$  in the direction of the current is much larger than the transverse field  $E_H$ .

(d)  $I = qnAv$ , and we assume that  $q = e$ . So we have

$$n = \frac{I}{eAv} = \frac{0.3 \text{ ampere}}{(1.6 \times 10^{-19} \text{ C})(0.08 \text{ m} \times 0.012 \text{ m})(4.8 \times 10^{-3} \text{ m/s})} = 4 \times 10^{23} \text{ carriers/m}^3$$

This is a very low density of charge carriers. The density of free electrons in copper is  $8 \times 10^{28}$  per  $\text{m}^3$ . Evidently this slab of material is not an ordinary metal: the charge carriers are positive, and the density of charge carriers is very low.

(e) The potential difference along 15 cm of material is 0.73 volts =  $RI$ , so

$$R = \frac{\Delta V}{I} = \frac{0.73 \text{ volts}}{0.3 \text{ ampere}} = 2.4 \text{ ohms}$$