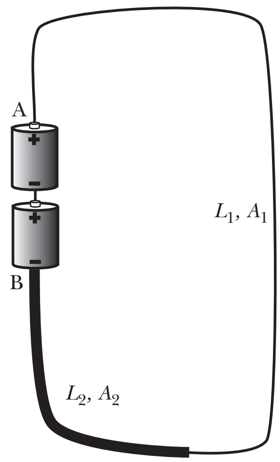
18.RQ.41

Experiment	**Effect on current**	**Parameter(s) that changed**
Double the length of a Nichrome wire	$\frac{i}{2}$	n A u ***E*** E/2
Double the cross-sectional area of a Nichrome wire	$2i$	n ***A*** u E A×2
Two identical bulbs in series compared to a single bulb	about $0.7i$	n A ***u*** ***E*** E/2; *u* bigger
Two batteries in series compared to a single battery	$2i$ (for Nichrome wire)	n A u ***E*** E×2

18.P.50 Two Nichrome wires



Thin wire: $L_1 = 0.5\text{ m}$

$$A_1 = \pi\left(\frac{0.25\times10^{-3}\text{ m}}{2}\right)^2$$

Thick wire: $L_2 = 0.15\text{ m}$

$$A_2 = \pi\left(\frac{0.35\times10^{-3}\text{ m}}{2}\right)^2$$

(a) <u>Node rule:</u> In steady state, current in thick and thin wires must be equal:

$$nA_1uE_1 = nA_2uE_2\text{, so } E_2 = \left(\frac{A_1}{A_2}\right)E_1$$

<u>Loop rule:</u> Round-trip potential difference = 0:

$$2(1.5\text{ V}) - E_1L_1 - E_2L_2 = 0\text{, so } E_1\left(L_1 + \left(\frac{A_1}{A_2}\right)L_2\right) = 3\text{ V}$$

Solving for E_1 and E_2:

$$E_1 = \frac{3\text{ V}}{\left(0.5\text{ m} + \left(\frac{0.25}{0.35}\right)^2(0.15\text{ m})\right)} = 5.20\text{ V/m}\text{ , } E_2 = \left(\frac{A_1}{A_2}\right)E_1 = \left(\frac{0.25}{0.35}\right)^2(5.2\text{ V/m}) = 2.65\text{ V/m}$$

(b)

Thin wire: $v_1 = uE_1 = \left(7\times10^{-5}\frac{\text{m/s}}{\text{N/C}}\right)(5.20\text{ N/C}) = 3.6\times10^{-4}\text{ m/s}$

Thick wire: $v_2 = uE_2 = \left(7\times10^{-5}\frac{\text{m/s}}{\text{N/C}}\right)(2.65\text{ N/C}) = 1.9\times10^{-4}\text{ m/s}$

$$\text{time} = \left(\frac{0.5\text{ m}}{3.6\times10^{-4}\text{ m/s}}\right) + \left(\frac{0.15\text{ m}}{1.9\times10^{-4}\text{ m/s}}\right) = 2200\text{ s} = 36\text{ minutes}$$

(c) Electric field propagates at about 1 ft/ns, or 0.3 m/ns. The shift in the electron sea takes very little time because each electron moves a very small distance, so the only significant time is the time it takes is for the change in E to propagate 0.5 m, or about 1.5 ns.

(d) $i_1 = nA_1v_1 = \left(9\times10^{28}\frac{\text{electrons}}{\text{m}^3}\right)\pi\left(\frac{0.25\times10^{-3}\text{ m}}{2}\right)^2(3.6\times10^{-4}\text{ m/s}) = 1.6\times10^{18}\text{ electrons/s}$

In the steady state the current is the same throughout the entire series-connected circuit.

18.P.51 Different mobilities
(a) Bulbs: $A_1 = A_2 = A$, $L_1 = L_2 = L$, $n_1 = n_2 = n$, $3u_1 = u_2$

Node rule: $i_2 = i_1$, so

$$nAu_1E_1 = nAu_2E_2 = nA(3u_1)E_2$$

$$E_2 = \frac{E_1}{3}$$

(b) In circuit (a), the loop rule (energy conservation) gives the following in the approximation that $\Delta V_{\text{wires}} \approx 0$ (since the wires are thick and have high mobility compared to the bulb filaments):

$$2\text{emf} - E_1L - \left(\frac{E_1}{3}\right)L = 0\text{, so } E_1 = \frac{3}{4}\left(\frac{2\text{emf}}{L}\right)$$

$$\text{In circuit (b), } 2\text{emf} - E_0L = 0\text{, so } E_0 = \frac{2\text{emf}}{L}$$

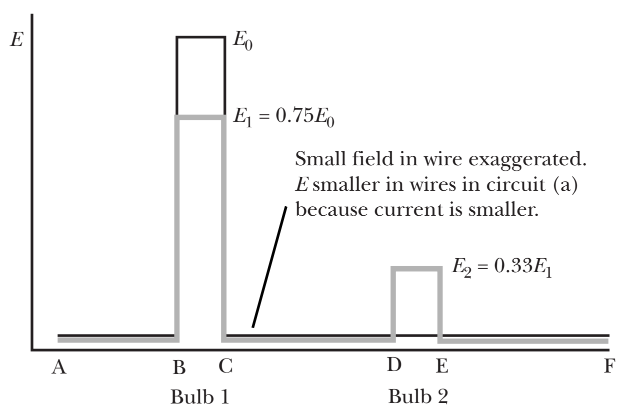
Same batteries, so $E_0 = \frac{4}{3}E_1$, and $v_0 = \frac{4}{3}v_1$ if we make the simplifying assumption that the mobility is unchanged despite the different current (and therefore different temperature).

So we can relate the current i_1 in circuit (a) to the current i_0 in circuit (b):

$$i_1 = nAv_1 = nA\left(\frac{3}{4}v_0\right) = \frac{3}{4}i_0$$

(c) Because the mobility u is very high for Cu, only a very small electric field is required to get a large drift speed. Since in the steady state the current through the Cu wires must be the same as the current through the bulb, and since the Cu wires have large cross-sectional area, only a small drift speed is needed in the Cu wires. So E can be very, very small in the connecting wires. Since all the Cu wires have the same cross-sectional area, the same current in all of them ⇒ the same drift speed ⇒ the same E.

(d) Gray lines are for circuit with two bulbs (part a):

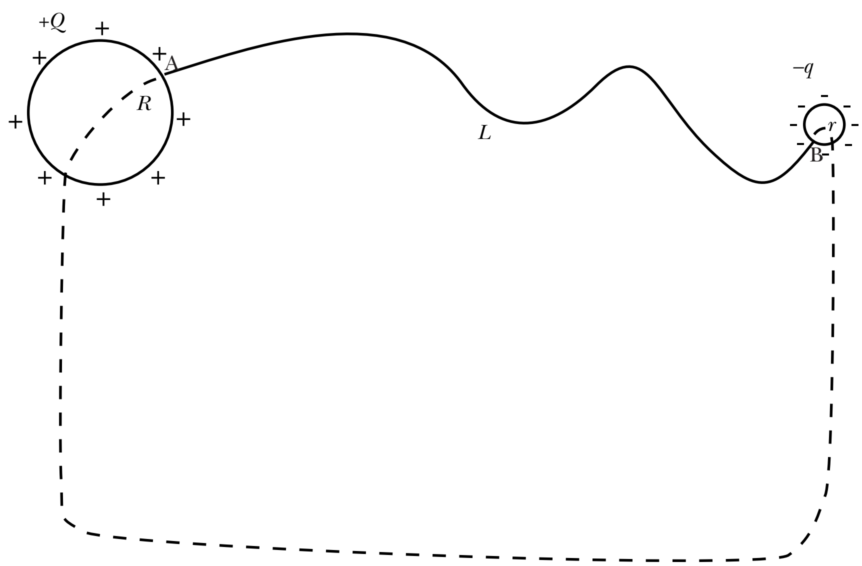


18.P.56 Connect two charged spheres together

Since this system takes a very long time to come to static equilibrium, we can consider it to be in a quasi steady state. That is, at any instant we can assume electron current i is the same at all locations in the wire. We are interested in the electron current in the wire a short time Δt after the connection has been made–long enough for the rearrangement of surface charge to have occurred (this takes only nanoseconds), but short enough that the charge on the two spheres has changed very little.

Since $i = nAuE$, and we know n, A, and u, we need to find the electric field E in the wire. There are three sources of charge contributing to E in the wire: the two charged spheres, and the charge on the surface of the wire. Although there is not a lot of charge on the wire, this charge is very close to the wire, and its contribution to E inside the wire is significant. Since we don't know the exact amount and distribution of charge on the wire, we can't calculate E directly from Coulomb's law.

Therefore, we start (as usual) from a fundamental principle: $\Delta V = 0$ for a round trip path. We choose a path that goes through the wire, where we want to know E, and also through the air, as shown in the diagram.



Consider the part of the path indicated by a dotted line. Along the sections of the path inside the spheres, the electric field is zero, and hence ΔV is zero.

Along the section of the path in the air, if we assume that (1) we are sufficiently far from the wire that the small amount of charge on the wire contributes a negligible electric field, and (2) the charged spheres are sufficiently far from each other that the electric field of one is negligible near the other, then we find that along the dotted line portion of the path:

$$\Delta V_{air} = V_B - V_A \approx \frac{1}{4\pi\varepsilon_0}\left(\frac{-q}{r} - \frac{Q}{R}\right)$$

So for a round trip path from A back to A, we have: $\Delta V = 0 = \Delta V_{wire} + \Delta V_{air} = \Delta V_{wire} + \frac{1}{4\pi\varepsilon_0}\left(\frac{-q}{r} - \frac{Q}{R}\right)$.

Since the system is in a quasi steady state, E must be uniform in the wire, so $\Delta V_{wire} = EL$ (it is positive, since we are going from B to A, travelling opposite to the direction of E in the wire). Thus,

$E = \dfrac{\frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r} + \frac{Q}{R}\right)}{L}$ and $i = nAu\left(\dfrac{\frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r} + \frac{Q}{R}\right)}{L}\right)$, and therefore $nAu\left(\dfrac{\frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r} + \frac{Q}{R}\right)}{L}\right)\Delta t$ electrons leave the small sphere in a time Δt.