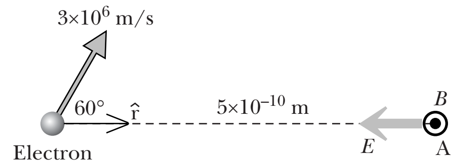
17.P.43 Fields of an electron

(a) Electric field points toward the negative electron.
Magnetic field is out of the page in direction of $(-e)\vec{v} \times \hat{r}$.

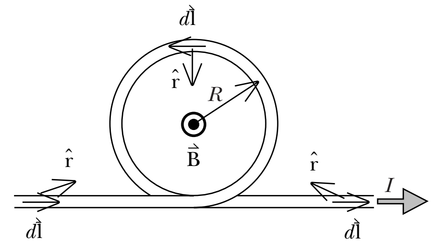


(b) $$E = \frac{1}{4\pi\varepsilon_0}\frac{|-e|}{r^2} = \left(9\times10^9\,\frac{\mathrm{N\cdot m^2}}{\mathrm{C^2}}\right)\frac{(1.6\times10^{-19}\,\mathrm{C})}{(5\times10^{-10}\ \mathrm{m})^2} = 5.8\times10^9\,\frac{\mathrm{N}}{\mathrm{C}}$$

$$B = \frac{\mu_0}{4\pi}\frac{|(-e)\vec{v}\times\hat{r}|}{r^2} = \frac{\mu_0}{4\pi}\frac{ev\sin(60^\circ)}{r^2} = \left(10^{-7}\,\frac{\mathrm{T\cdot m}}{\mathrm{A}}\right)\frac{(1.6\times10^{-19}\,\mathrm{C})(3\times10^{6}\ \mathrm{m/s})\sin(60^\circ)}{(5\times10^{-10}\ \mathrm{m})^2} = 0.17\ \mathrm{T}$$

17.P.48 Wire with a loop in it

The magnetic field at the center of the ring is the sum of the magnetic field of the straight wire and the magnetic field of the loop. Both the wire and the loop make B out of the page, so the net field points out of the page.



$$\vec{B}_{\text{loop}} = \int \frac{\mu_0}{4\pi} \frac{I d\vec{l} \times \hat{r}}{r^2} \text{ and } d\vec{l} \perp \hat{r} \text{ everywhere on the loop, so}$$

$$B_{\text{loop}} = \frac{\mu_0}{4\pi} \frac{I}{r^2} \int dl \sin 90° = \frac{\mu_0}{4\pi} \frac{I}{r^2}(2\pi r) = \frac{\mu_0}{4\pi} \frac{2\pi I}{r} \text{ out of the page}$$

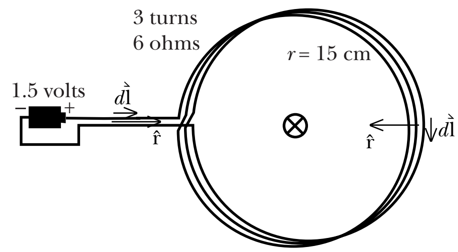
(special case of on-axis field of loop)

$$B_{\text{straight wire}} = \frac{\mu_0}{4\pi} \frac{2I}{r} \text{ out of the page}$$

$$B_{\text{net}} = \frac{\mu_0}{4\pi} \frac{2I}{r}(1 + \pi) \text{ out of the page}$$

17.P.49 Deflecting a compass needle with a coil

First we need to find the magnetic field due to the current in the circuit, at the position of the compass (center of the loops).



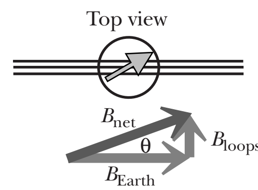
The straight segments of wire make a negligible contribution, since for them $d\vec{l} \times \hat{r} \approx 0$.

The magnetic field due to a representative segment of the loop is into the page, at the center of the loop. All segments of the loop contribute $d\vec{B}$ in the same direction. The magnitude B due to all $N = 3$ loops of radius $R = 15$ cm is this:

$$B_{\text{loops}} = \int N\frac{\mu_0}{4\pi}\frac{Idl\sin(90°)}{R^2} = N\frac{\mu_0}{4\pi}\frac{I}{R^2}\int dl = N\frac{\mu_0}{4\pi}\frac{I}{R^2}(2\pi R) = N\frac{\mu_0}{4\pi}\frac{2\pi I}{R}$$

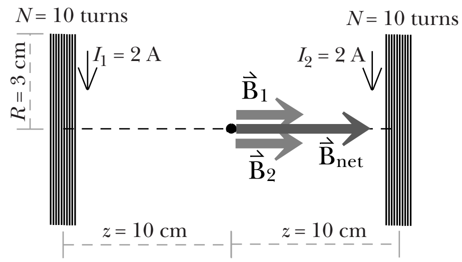
$$B_{\text{loops}} = (3)\left(10^{-7}\frac{\text{T}\cdot\text{m}}{\text{A}}\right)\frac{2\pi(0.25\text{ A})}{(0.15\text{ m})} = 3\times10^{-6}\text{ T}$$, into the page, due to current-carrying loops

Second, we need to calculate the effect on the compass. Looking down from above, the compass deflects inward by this amount:



$$\theta = \arctan\left(\frac{B_{\text{loops}}}{B_{\text{Earth}}}\right) = \arctan\left(\frac{3\times10^{-6}\text{ T}}{2\times10^{-5}\text{ T}}\right) = 8.5°$$

17.P.52 Magnetic field of coils



(a) Both coils make a magnetic field in the same direction at the point of interest. Since the point is halfway between them, the magnitudes of the contributions are also the same:

$$B_1 = B_2 = N\frac{\mu_0}{4\pi}\frac{2\pi R^2 I}{(z^2 + R^2)^{3/2}}$$

$$B_{net} = B_1 + B_2 = N\frac{\mu_0}{4\pi}\frac{4\pi R^2 I}{(z^2 + R^2)^{3/2}} = (10)\left(10^{-7}\frac{\text{T}\cdot\text{m}}{\text{A}}\right)\frac{4\pi(0.03\text{ m})^2(2\text{ A})}{[(0.1\text{ m})^2 + (0.03\text{ m})^2]^{3/2}} = 2\times10^{-5}\text{ T}$$

(b) Using the $1/z^3$ approximation:

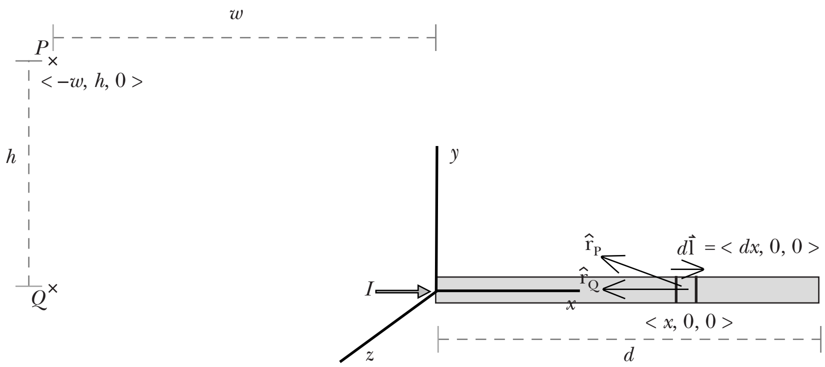
$$B_{net} \approx N\frac{\mu_0}{4\pi}\frac{4\pi R^2 I}{z^3} = (10)\left(10^{-7}\frac{\text{T}\cdot\text{m}}{\text{A}}\right)\frac{4\pi(0.03\text{ m})^2(2\text{ A})}{(0.1\text{ m})^3} = 2.3\times10^{-5}\text{ T}$$

$$\frac{2.3}{2.0} = 1.15\text{, so there is a 15\% error}$$

(c) If the current in the right loop is reversed, the magnetic field B_2 now points to the left. Since it is still equal in magnitude to B_1, the net field is zero at the midpoint.

17.P.55 Magnetic field of a current

(a) At location Q, $d\vec{l} \times \hat{r}_Q = 0$, so $\vec{B}_Q = 0$.



(b) Divide wire into slices as shown on diagram.

$$\hat{r}_P = \frac{<-w, h, 0> - <x, 0, 0>}{r} = \frac{<(-w-x), h, 0>}{[(w+x)^2 + h^2]^{1/2}}$$

$$d\vec{B} = \frac{\mu_0}{4\pi} I \frac{<dx, 0, 0> \times <(-w-x), h, 0>}{[(w+x)^2 + h^2]^{3/2}}$$

$$d\vec{B} = \frac{\mu_0}{4\pi} I \frac{<0, 0, h dx>}{[(w+x)^2 + h^2]^{3/2}}, \text{ so } B_x = B_y = 0$$

$$B_z = \frac{\mu_0}{4\pi} I h \int_0^d \frac{dx}{[(w+x)^2 + h^2]^{3/2}}$$

(c) Direction is out of page, in $+z$ direction (see cross product calculation above).