Physics 432/750: Cosmology
Winter 2003-2004
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Problem Set 3

Due Thursday, February 19.

1. Energy densities

If $\Omega_{matter} = 0.3, \Omega_{vac} = 0.7$ and $T_{CMB} = 2.73$ K, at what redshift was the radiation energy density (really, the density of relativistic particles, which includes both photons and neutrinos) comparable to the vacuum energy density?

2. Epoch of Matter-Radiation Equality

(a) Derive an approximate relation for the redshift of matter-radiation equality, by extrapolating backwards from the matter and radiation densities at the present epoch.

(b) If a significant fraction of the primordial mass density is comprised of particles that decay into relativistic species at early times, how does this effect the epoch of matter-radiation equality? (Hint: remember that the relevant density of "radiation" for computing the epoch of matter-radiation equality includes the energy density in particles that are relativistic at that time.)

3. Entropy of radiation

Show that the first law of thermodynamics

$$dE = TdS - PdV$$

leads to

$$S = \frac{4aT^3V}{3}$$

for thermal radiation.

4. Nucleosynthesis

Show that the the helium mass fraction is

$$Y = 2\left(1 + \frac{n_p}{n_n}\right)^{-1}$$

if all the baryons are in H and He.