Physics 432/750: Cosmology
Winter 2003-2004
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Problem Set 1

Due Thursday, January 22.

1. Robertson-Walker metric
Show that the Robertson-Walker metric

$$c^2 d\tau^2 = c^2 dt^2 - R^2(t)[dr^2 + S_k^2(r) d\psi^2]$$

can also be written in the form

$$c^2 d\tau^2 = c^2 dt^2 - R^2(t)[dr^2/(1 - kr^2) + r^2 d\psi^2]$$

2. Metric for an Open Universe
For a $k = -1$ Friedmann cosmology ($\Lambda = 0$), with $\rho = p = 0$, show that the R-W metric line element becomes

$$c^2 d\tau^2 = c^2 dt^2 - c^2 t^2 [dr^2 + \sinh^2 r d\psi^2]$$

3. Relativistic Velocities

(a) Show that the general relativistic relation between recession velocity and cosmological redshift is

$$v_{rec}(t, z) = \frac{c}{R_0} \dot{R}(t) \int_0^z \frac{dz'}{H(z')}$$

where $H(z')$ is the Hubble constant at redshift z'.

(b) Show that the special relativistic relation between peculiar velocity and redshift is

$$v_{pec}(z) = c \frac{(1 + z)^2 - 1}{(1 + z)^2 + 1}$$

(c) Show that both relativistic relations are approximately $v \approx cz$ at small distance.

4. Particle Horizon
Following the suggestions outlined on pp. 85-86 of the text, show that the dominant form of mass-energy at early times must scale as $\rho \propto R^{-\alpha}$ with $\alpha > 2$ for a particle horizon to exist.

5. Evolution of $\Omega_{matter}, \Omega_{vac}$
Compute how the density parameters Ω_{matter} and Ω_{vac} evolve with time in different cosmologies and plot the results on a figure like 3.5 on p. 83. In other words, for each choice of $\Omega_{matter}, \Omega_{vac}$ today, plot the trajectory of the model backwards in time to where it would lie in that same diagram at $t \approx 0$. On the plot, clearly indicate which end of each curve is now and which is at $t = 0$. Pick models from all regions of the diagram and be sure to include the following (in other words, do more than just these):
$\Omega_{matter} = 1, \Omega_{vac} = 0$
$\Omega_{matter} = 0.2, \Omega_{vac} = 0$
$\Omega_{matter} = 0.3, \Omega_{vac} = 0.7$