PHYS 305 - Assignment #4

Make sure your name is listed as a comment at the beginning of all your work.

Purpose: Develop a physical intuition for the organization of a chaotic attractor, the Lorenz butterfly chaotic attractor.

Fixed Points

- Find analytically the fixed points of the Lorenz model.
- Calculate (numerically) the locations of the fixed points for the model parameters yielding the butterfly attractor, i.e., $\sigma = 10$, $b = 8/3$ and $R = 28$.
- Plot the fixed points in the same graph as the butterfly attractor.
- Comment on what you found.

It is an attractor!

As the name indicates the butterfly chaotic attractor is *an attractor*. This means that trajectories originating in the basin of attraction will converge, after a transient period, to the butterfly attractor.

You will illustrate this effect by launching 6,400 trajectories from the surface of a sphere of large radius ($R = 55$) distributed uniformly on the surface of the sphere. To do so use the following coordinate transform.

$$\theta = 2\pi u$$

$$\phi = acos(2v - 1)$$

- Set up a 2-D 80x80 equally spaced square lattice (6,400 points) in u and v coordinates.
- Calculate the corresponding angles θ and ϕ for each values of u and v.
- Position the sphere at the geometrical center of the butterfly attractor. The latter should be computed as the *pseudo center of mass* of the attractor, calculated assuming a mass of one unit for each point along the trajectory that traces the butterfly attractor.
- Calculate the x, y and z coordinates as if the angles θ and ϕ were spherical coordinates angles, using a radius $R = 50$.
- Feed these calculated points on a sphere as launch points for (6,400) trajectories calculated by the lorenz.c code. Use $t_{max} = 200$ and no warm-up. Record the *last point of each trajectory* (at the time=200) and plot these in 3-D.

Now, isn't this a surprise!!

- What shape do you get?
- Superpose the butterfly chaotic attractor (obtained from ONE trajectory!) onto the shape just obtained.
- Write an analysis code that will calculate the furthest distance reached on the last time slice of these 6,400 trajectories from the geometrical center of the butterfly chaotic attractor.
- Refine the shape by using a u-v 100x100 (10,000 points) grid.

Order - really?

Lorenz in his original seminal work observed that *a trajectory leaves one spiral only after exceeding some critical distance from its center*. Moreover, the extent to which this distance is exceeded appears to determine the point at which the next spiral is entered; this in term seems to determine the number of circuits to be executed before changing spirals again. This implies that a maximum of z suffices to predict the next maximum of z.

- Write an analysis code that finds all the maxima and/or minima of $z(t)$. The code should decide what to calculate based on line arguments that could be $\langle -\text{max} \rangle$ and/or $\langle -\text{min} \rangle$. The code should then read the result of solving the Lorenz model from $\langle \text{stdin} \rangle$. It should detect the maxima (minima) in $z(t)$ and then calculate the exact value of each maximum (minima) via a three points parabola interpolation based on function values at three adjacent points on the numerical lattice dt apart.
- Plot adjacent maxima of $z(t)$ versus each other, i.e., z_{max}^{k+1} versus z_{max}^{k} for all k.
- Repeat the plot above, this time plotting minima versus adjacent minima.

Poincare Surface of Section

A *Poincare Surface of Section* records the crossing of a trajectory with an arbitrarily chosen plane. This builds a 2-dimensional *map* based on the location of the sequential crossings in the plane.

- Plot $z(t)$ versus t from the *Lorenz Attractor* over some small time domain. Note the complexity of this graph.
- Write a program to calculate the *Poincare Section* of the *Lorenz Attractor*. Define this *Poincare Surface of Section* as the intersection of the trajectory with the plane $z = 37.0$. The code should find when the trajectory crosses the plane and then use linear interpolation to define precisely the location of the intersections with the plane.

- Produce two images: one that captures all crossings with the plane and a second that records only to the crossings from below to above the plane.

The *Lorenz Butterfly Attractor* is a *fractal object.* Yet, there is order in this chaotic solution!