PHYS 305 - Assignment #5
Due: Friday, March 11, 2016

This is a fun project that will illustrate how a simple Harmonic Oscillator can lead to complicated behavior when driven and damped.

Purpose: Develop a physical intuition for a damped and driven system.

Duffing Oscillator

The *Stroboscopic Projection* of the Duffing Oscillator solution in the chaotic regime generates an image of the *Chaotic Attractor*. We used in class the helper code *stroboscopic.c* that generated the data to plot.

Part 1 − Screen Definition

Generate images of the Duffing 1-well and 2-well Chaotic Attractors with various definition, i.e., various No of points: 100, 500, 2,000 and 5,000.

Comment on what you see.

Part 2 − Projection

We projected the x and v values at the time t equal to integer multiples of the period T of the external driving force. This was a natural and convenient choice, but by no mean unique one.

Form images (use 1000 points) of the 2-well Duffing Chaotic Attractor based on the projections at
$t_n = (n + 1/4)T$
$t_n = (n + 1/2)T$
$t_n = (n + 3/4)T$

Part 3 − Shadowing Effect

The plot of the 2-well Duffing Chaotic Attractor results from solving a non-linear system of differential equations. Therefore the solution should depend sensitively on initial conditions.

So one could think that the solution might be *thrown-off* by the lack of accuracy of the RK4 solver.

So you are asked to check if this occurs!

- Generate the data for the 2-well Duffing Chaotic Attractor using 1000 points.

- *Change dt* to a value twice as big in the duffing code. Check that *dt* might already be a parameter on the command line.

- Redo the calculation of the 2-well Duffing Chaotic Attractor and store the data

- Plot (with dots) the 2-well Duffing Chaotic Attractor calculated both ways in a single plot.

- How do they compare?

- *But wait!* Plot (in a single plot and with dots) x vs t over a small time domain (using *set xrange=[0:150]*) for the two calculations. Use **the data from the Duffing code, i.e., before the stroboscopic projection**

- Do they agree or disagree? In what way?

- Explain what is going on?

Part 4 − Movie

Make a *movie* based on the build-up of the 2-well Duffing Chaotic Attractor as you draw it with an ever increasing the number of points. Start with 10 points and end with 4000 points.

The increase in the number of points in going from frame to frame is not required to be a constant. The overall goodness criteria is to have a good looking and smooth movie.

You can *produce this movie in whatever way you want*

One easy way is to store the data in a subdirectory, and run a script (commands for gnuplot) to instruct gnuplot to *plot & replot* the fractal with more and more points.