The “eye-curves” of Strange Attractors

Timothy Jones, Daniel Cross, Robert Gilmore (Physics, Drexel University)

Every chaotic system is governed by a set of mathematical equations. These equations are deterministic: the motion they predict is not random. Since the work of Poincaré, mathematicians and physicists have made much progress in understanding such systems. The key is to find invariants that can indicate the properties that the system will exhibit both globally and locally. In this poster, we present research our group has been working on in trying to understand a type of property in chaotic systems we call the eye-curve. The eye-curve seems to "organize" the flow of chaotic attractors much like the eye of a hurricane or tornado "organizes" the behavior of those systems. We show the eye-curve for a number of strange attractors and suggest their possible application in the analysis of chaotic systems.

The Theoretical

Dynamical systems can often be written in the form,

$$\mathbf{V} = \frac{d\mathbf{x}}{dt} = \mathbf{f}(\mathbf{x}), \quad \mathbf{x} \in \mathbb{R}^n, \ \mathbf{x}(0) = \mathbf{x}_0$$

We can write the next derivative à la Fréchet:

$$\frac{d\mathbf{V}}{dt} \equiv \gamma = \frac{d\mathbf{f}(\mathbf{x})}{d\mathbf{x}}\frac{d\mathbf{x}}{dt} = \mathbf{Jf}(\mathbf{x})$$

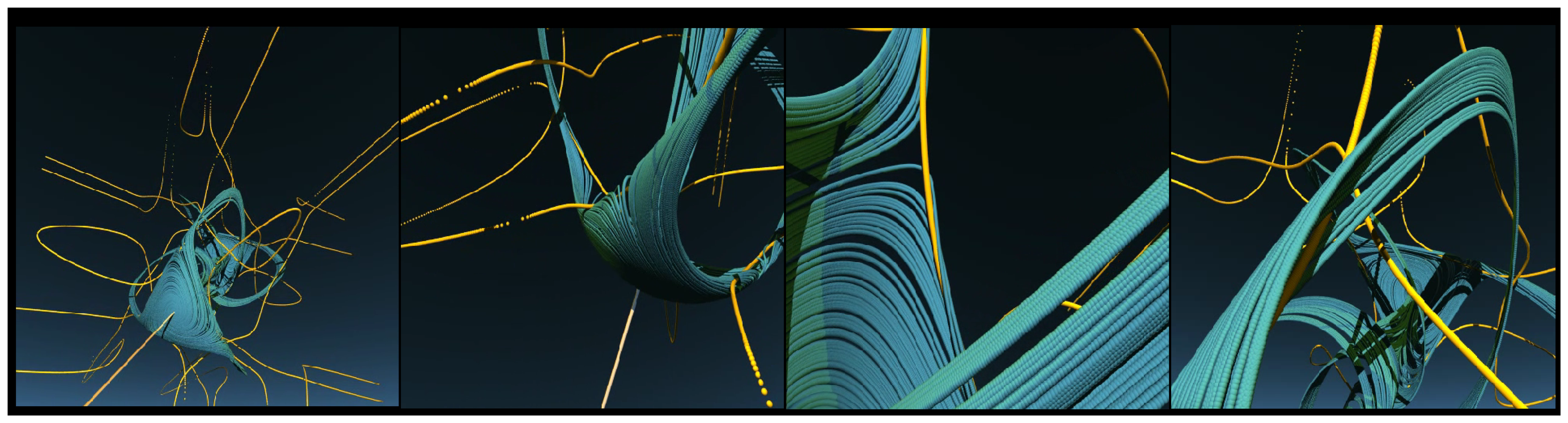
We now form an eigenvalue equation:

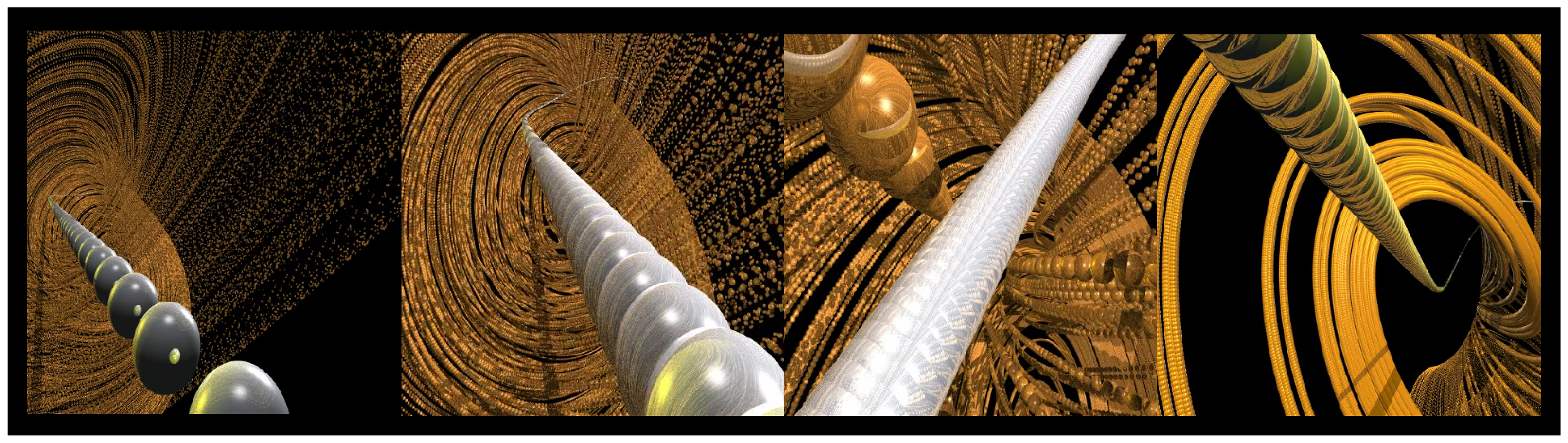
$$\mathbf{Jf}(\mathbf{x}) = \lambda \mathbf{f}(\mathbf{x})$$

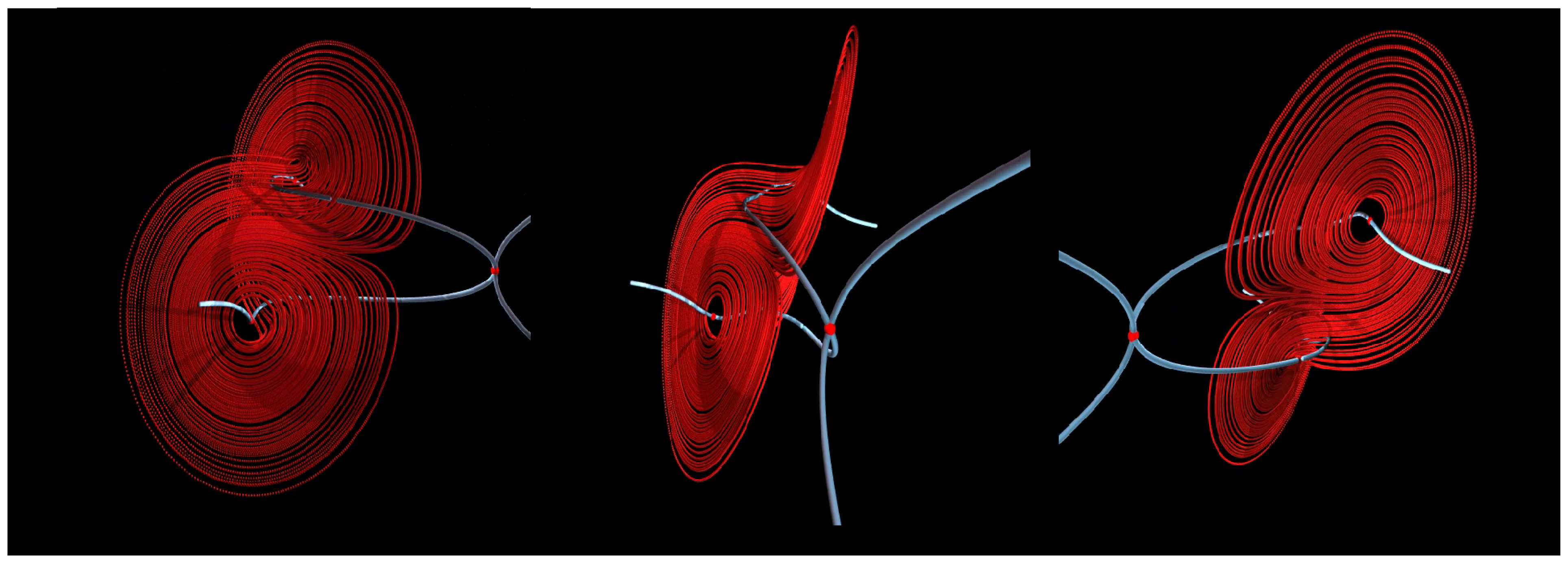
$$\begin{pmatrix} \frac{\partial f_1(x)}{\partial x_1} & \frac{\partial f_1(x)}{\partial x_2} & \frac{\partial f_1(x)}{\partial x_3} \\ \frac{\partial f_2(x)}{\partial x_1} & \frac{\partial f_2(x)}{\partial x_2} & \frac{\partial f_2(x)}{\partial x_3} \\ \frac{\partial f_3(x)}{\partial x_1} & \frac{\partial f_3(x)}{\partial x_2} & \frac{\partial f_3(x)}{\partial x_3} \end{pmatrix} \begin{pmatrix} f_1(x) \\ f_2(x) \\ f_3(x) \end{pmatrix} = \lambda \begin{pmatrix} f_1(x) \\ f_2(x) \\ f_3(x) \end{pmatrix}$$

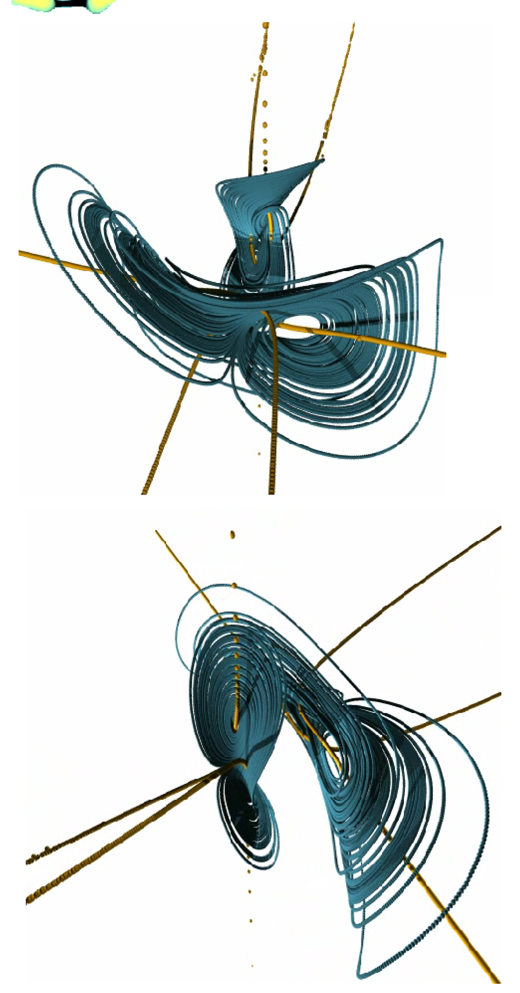
We have found that the set $\{\lambda : \Im(\lambda) = 0\}$ form a curve through the three dimensional strange attractors we have studied.

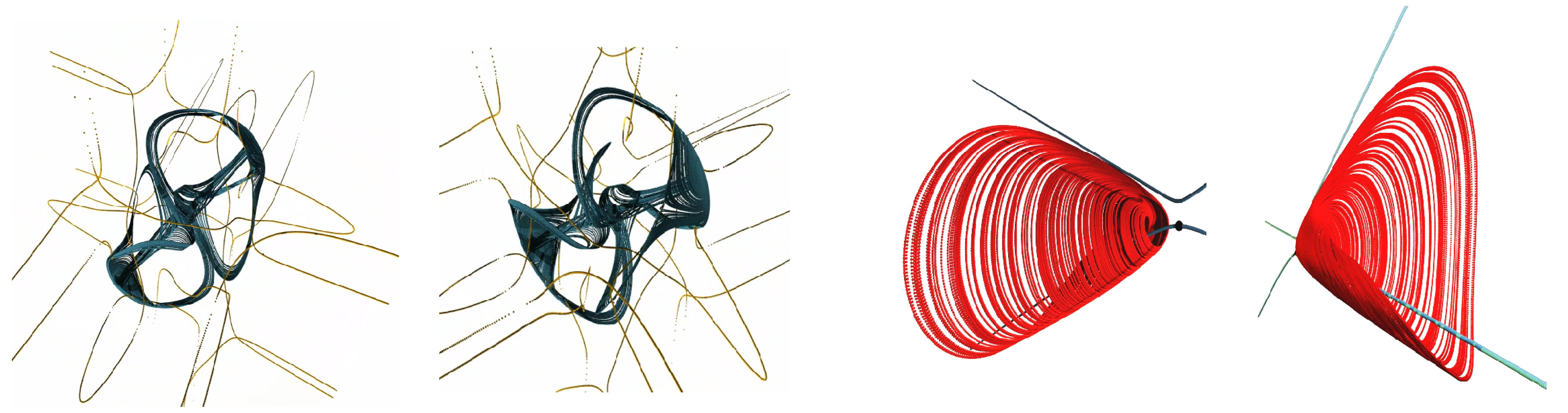
We are currently working to understand how these curves fit into the context of the chaos program using tools of differential geometry.











The Numerical

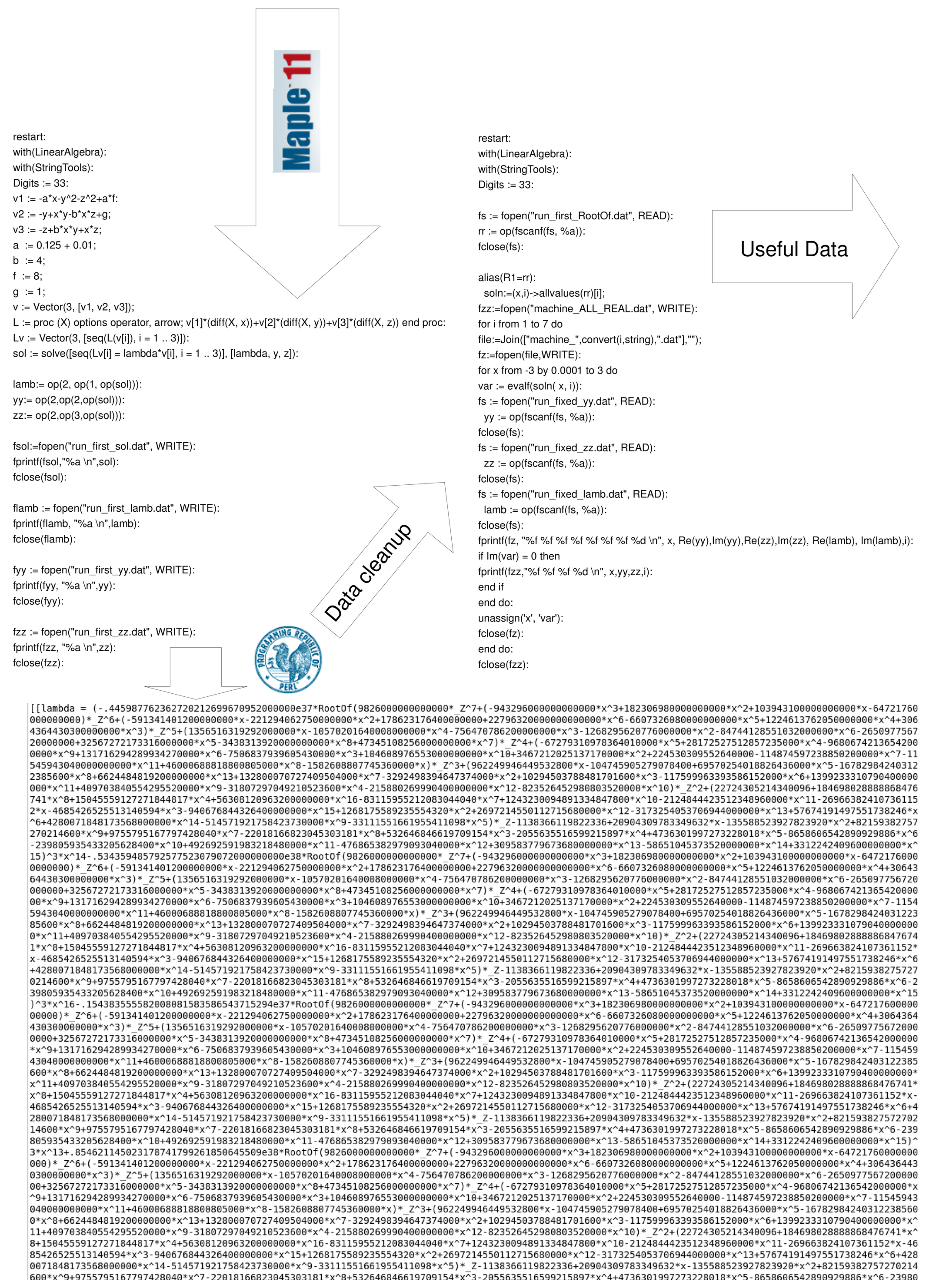
Lorenz 84 Attractor

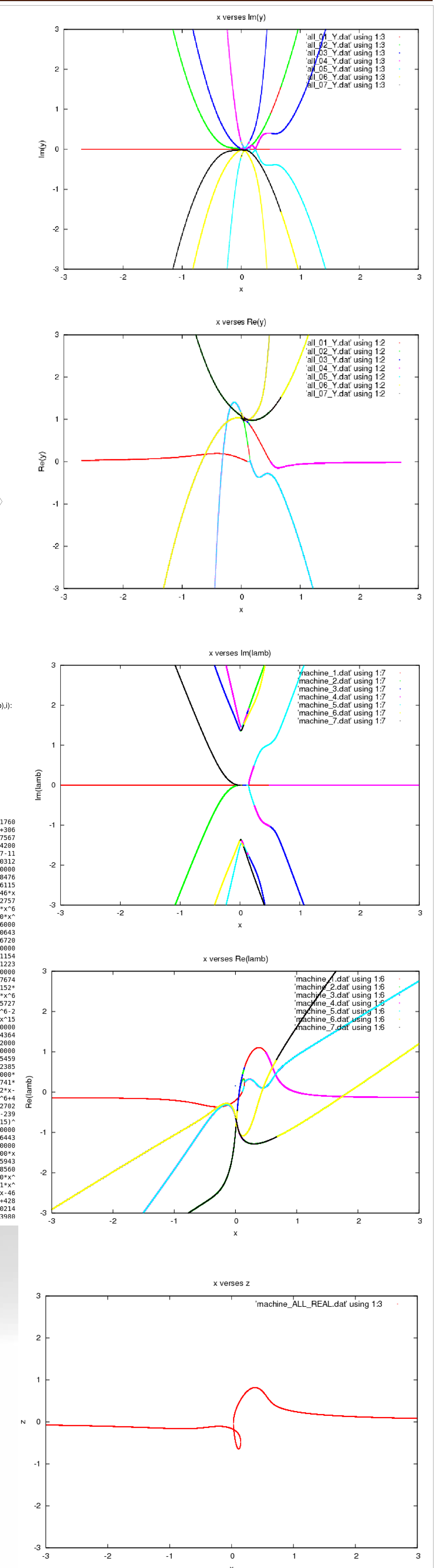
This system is defined by the set of equations:

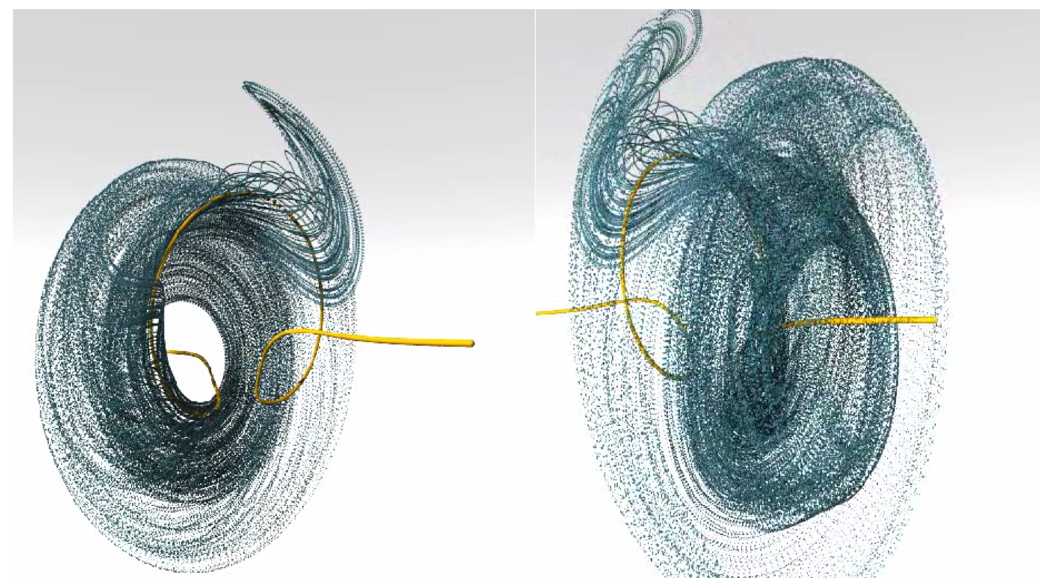
$$\begin{aligned} \frac{dx}{dt} &= -ax - y^2 - z^2 + af \\ \frac{dy}{dt} &= -y + xy - bxz + g \\ \frac{dz}{dt} &= -z + bxy + xz \end{aligned}$$

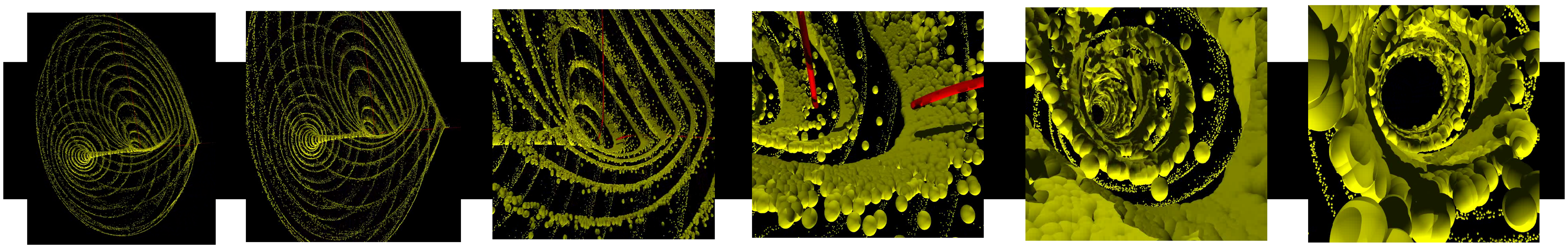
We used values $a = 0.25, b = 4, f = 8$, and $g = 1$. We then calculate $J_{ij} = \partial f_i(x)/\partial x_j$ and the eigenvalue equation $J_{ij} f_j = \partial f_i(x)/\partial x_j f_j = \lambda f_i$ using Maple. Explicitly,

$$\begin{pmatrix} \frac{\partial f_1(x)}{\partial x_1} & \frac{\partial f_1(x)}{\partial x_2} & \frac{\partial f_1(x)}{\partial x_3} \\ \frac{\partial f_2(x)}{\partial x_1} & \frac{\partial f_2(x)}{\partial x_2} & \frac{\partial f_2(x)}{\partial x_3} \\ \frac{\partial f_3(x)}{\partial x_1} & \frac{\partial f_3(x)}{\partial x_2} & \frac{\partial f_3(x)}{\partial x_3} \end{pmatrix} \begin{pmatrix} f_1(x) \\ f_2(x) \\ f_3(x) \end{pmatrix} = \lambda \begin{pmatrix} f_1(x) \\ f_2(x) \\ f_3(x) \end{pmatrix}$$









See http://www.physics.drexel.edu/~tim for films of these simulations.