Name: ________________________________

You may answer the questions in the space provided here, or if you prefer, on your own notebook paper.

Short answers

1. $\boxed{\text{2 points}}$ Why might your cell phone signal actually go down if your cell provide built a new antenna near you? (Ignore the various corrections engineers have for this, just assume the antennas are simple single band-width broadcasters).

   Solution: Interference patterns if the geometrical location is just right could result in destructive interference between the new antenna and others you may have been using before.

2. $\boxed{\text{2 points}}$ Why don't ocean waves entering the Delaware Bay experience diffraction? (You may Google-Maps Delaware Bay for perspective).

   Solution: The bay opening is too wide compared to the wavelength of waves.

3. $\boxed{\text{2 points}}$ What are the two principles of relativity?

   Solution:

   1. There is no preferred reference frame; the laws of physics are the same in all frames.
   2. It is a law of physics that the speed of light is $c = (2.99 \times 10^8)\,\frac{\mathrm{m}}{\mathrm{s}}$.

4. $\boxed{\text{2 points}}$ Which additional principle was key to special relativity.

   A. There is a universal time.

   B. There is a universal reference frame.

   C. The speed of light is constant in all reference frames.

   D. The speed of light is constant in frames at rest.

5. $\boxed{\text{2 points}}$ Proper time is

A. Found in the frame at rest relative to the Universe.

B. Found in the frame in which the time of an incident would be the shortest.

C. Found in the frame in which the time of an incident would be the longest.

Problems

6. $\boxed{\text{10 points}}$ A radio station is allowed to broadcast at an average power not to exceed 25 kW. If the electric field amplitude of 0.020 V/m is acceptable for receiving the radio transmission, estimate how many kilometers away you might be able to hear this station?

Solution: First relate minimal intensity to minimal electric field:

$$\begin{aligned}
\bar{I} &= \frac{1}{2}\epsilon_0 c E_0^2 \\
&= \frac{1}{2}\left(\left(8.854187 \times 10^{-12}\mathrm{C}^2/\mathrm{Nm}^2\right)\right)\left(\left(2.99 \times 10^8\right)\frac{\mathrm{m}}{\mathrm{s}}\right)\left(0.020\frac{\mathrm{V}}{\mathrm{m}}\right)^2 \\
&= 5.31 \times 10^{-7}\frac{\mathrm{W}}{\mathrm{m}^2}
\end{aligned}$$

As we said in class, this intensity spreads out as the area of a spherical shell, so that the original power is divided by the area of a spherical shell $4\pi r^2$, and we have:

$$\frac{25 \times 10^3\ \mathrm{W}}{4\pi r^2} = 5.31 \times 10^{-7}\frac{\mathrm{W}}{\mathrm{m}^2} \quad \rightarrow \quad r \approx 61000\ \mathrm{m} = 61\ \mathrm{km}$$

7. $\boxed{\text{5 points}}$ What will be the mean lifetime of a muon as measured in the laboratory if it is traveling at $v = 0.60c$ with respect to the laboratory? Its mean lifetime at rest is 2.20 μs. How far does it travel in the lab before decaying?

Solution: The proper frame for the lifetime of the muon is in the muon's frame. This is because we would measure the birth and death of the muon in the same spot in such a frame. In this frame, the lifetime is 2.20×10^{-6}s. In the lab frame, the muon's frame is moving at a speed of 0.60c. The lifetime a scientist in the lab would measure would be time-dilated as follows:

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{2.20 \times 10^{-6}\text{ s}}{\sqrt{1 - \frac{0.36c^2}{c^2}}} = 2.8 \times 10^{-6}\text{ s}$$

To find the distance in the lab frame, it is as simple as $d = v\Delta t$,

$$d = v\Delta t = 0.60\left(3.0 \times 10^8 \frac{\text{m}}{\text{s}}\right)\left(2.8 \times 10^{-6}\text{ s}\right) = 500\text{ m}$$

8. $\boxed{\text{5 points}}$ GPS satellites move at about 4000 m/s. Show that a good GPS receiver needs to correct for time dilation if it produces results consistent with atomic clocks accurate to 1 part in 10^{13}. *Hint, you will need the binomial expansion:* $1/\sqrt{1-x} \approx 1 + \frac{1}{2}x$ *when* x *is much smaller than* 1.

Solution: The proper time is in the satellite frame, so that relative to somebody on Earth,

$$\Delta t = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\Delta t_0 = \frac{1}{\sqrt{1-\left(\frac{4\times 10^3 \text{ m/s}}{3\times 10^8 \text{ m/s}}\right)}}\Delta t_0 = \frac{1}{\sqrt{1-1.8\times 10^{-10}}}\Delta t_0$$

Our calculators can't handle that, so we can use the binomial theorem, which applied to this equation yields:

$$\Delta t = \left(1 + \frac{1}{2}\left(1.8\times 10^{-10}\right)\right)\Delta t_0 = \left(1 + 9\times 10^{-11}\right)\Delta t_0$$

With out assuming relativity, we would get a time error of

$$\frac{(\Delta t - \Delta t_0)}{\Delta t_0} = 1 + 9\times 10^{-11} - 1 = 9\times 10^{-11}$$

That is about 1000 times greater than the atomic clocks accuracy of 1 part in 10^{13} which would introduce accumulating errors. There are also general relativity effects which must be accounted for.

9. $\boxed{\text{5 points}}$ When two moles of hydrogen and one mole of oxygen react to form two moles of water, the energy released is 484 kJ. Assuming (incorrectly, but for the sake of argument) that all this energy comes from the transform of mass to energy, how much does the mass decrease in this reaction?

Solution:

$$\Delta m = \frac{\Delta E}{c^2} = \frac{-484 \times 10^3 \text{ J}}{(3.00 \times 10^8 \text{ m/s})^2} = -5.38 \times 10^{-12} \text{ kg}$$

10. $\boxed{\text{5 points}}$ What is the kinetic energy and speed of an electron ejected from a sodium surface whose work function is $W_0 = 2.28$ eV when illuminated by light of wavelength 410 nm?

Solution: The energy of the photon is:

$$hf = \frac{hc}{\lambda} = 4.85 \times 10^{-19} \text{ J} = 3.03 \text{ eV}$$

The equation for the photo-electric effect is:

$$KE_{max} + W_0 = hf$$

So that in this case,

$$KE_{max} = 3.03 \text{ eV} - 2.28 \text{ eV} = 0.75 \text{ eV} = (0.75 \text{ eV}) \left(1.60 \times 10^{-19} \text{J/eV}\right) = 1.2 \times 10^{-19} \text{ J} = \frac{1}{2} m v^2$$

The mass of the electron is $m = 9.1 \times 10^{-31}$ kg so that we solve for $v_{max} = 5.1 \times 10^5$ m/s.

Some possibly useful equations.

$$n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$$

$$I = S_{avg} = \frac{E_{max}^2}{2\mu_0 c}$$

$$B = \frac{E}{c}$$

$$c = \lambda f$$

$$c^2 = a^2 + b^2$$

$$T = \frac{1}{f}$$

$$\mu_0 = \left(4 \times \pi \times 10^{-7}\mathrm{Tm/A}\right)$$

$$c = \left(2.99 \times 10^8\right) \frac{\mathrm{m}}{\mathrm{s}}$$

$$2nt = \left(m + \frac{1}{2}\right)\lambda, m = 0, 1, 2 \ldots$$

$$E = \frac{hc}{\lambda}$$

$$KE = E_{photon} - \phi$$

$$hc = 1240\mathrm{eV} \cdot \mathrm{nm}$$

$$E_{photon} = hf = \frac{hc}{\lambda}$$

$$m_p = 1.672 \times 10^{-27} \text{ kg}$$

$$q_p = 1.602 \times 10^{-19} \text{ C}$$

$$m_e c^2 = 0.511 \text{ MeV}$$

$$K = \gamma m c^2$$

$$K = E - mc^2$$

$$E^2 = p^2c^2 + (m_p c^2)^2$$

$$x' = \gamma(x - vt)$$

$$t' = \gamma(t - \frac{vx}{c^2})$$

$$L' = \frac{L_p}{\gamma}$$

$$t' = \gamma t_p$$

$$E = mc^2$$

$$\Delta E = \Delta mc^2$$

$$\bar{I} = \frac{1}{2}\epsilon_0 c E_0^2$$

$$\Delta t = \frac{\Delta t_p}{\sqrt{1 - \frac{v^2}{c^2}}}$$