Name: ______________________________

You may answer the questions in the space provided here, or if you prefer, on your own notebook paper.

Short answers

1. [5 points] What was Maxwell's finding that related light to electromagnetic waves?

Solution: Maxwell found that the speed of an EM wave was the same as the speed of light, suggesting that light itself was an EM wave (which we now know it is.

2. [5 points] List at least two things that suggest that light is a wave, and then list one key thing that suggest light is particle.

Solution: Light is a wave: Interference, speed is that of EM waves, refraction; **Light is particle:** Photoelectric effect

3. [5 points] What are the two principles of relativity?

Solution:

   1. There is no preferred reference frame; the laws of physics are the same in all frames.
   2. It is a law of physics that the speed of light is $c = (2.99 \times 10^8)\,\frac{\text{m}}{\text{s}}$.

4. [5 points] What radical realizations about space and time did relativity theory bring about?

Solution: That measurements of length (space) and time duration of events (time) differ between frames that are moving at a relative speed from each other, and that these differences increase as the relative speed between the frame increases. This suggest not only that time and space are connected in unexpected ways, but also that simultaneous events are not simultaneous in all frames.

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5. $\boxed{\text{5 points}}$ What happens when we shine a laser through a double slit apperatus? What will we see on a screen behind the laser? What happens if instead of a beam of light, we only send one photon at a time?

Solution: Light in the beam will interfere, and there will be an interference/diffraction pattern on the screen behind the double slit apparatus. If we send in one photon at a time, we will only see dots on the screen, but if we were to record that with a long-exposure camera, eventually billions of those dots would make up the same pattern as with the full beam–indicating that the photon is somehow interfering with itself.

Problems

6. $\boxed{\text{15 points}}$ A sinusoidal planer electromagnetic wave with frequency 30.0×10^6 Hz, in free space, traveling in the x direction has a maximum electric field amplitude of $150\hat{\mathbf{j}}$ N/C. Recall that in sinusoidal planer waves, the **E** and **B** fields are perpendicular to each other, and to the direction of motion.

   (a) Find the wavelength and period of the wave.

   Solution:
   $$\lambda = \frac{c}{f} = \frac{(2.99 \times 10^8)\,\frac{\mathrm{m}}{\mathrm{s}}}{30.0 \times 10^6 \times 10^6 Hz} = 9.97m$$
   and
   $$T = \frac{1}{f} = \frac{1}{30.0 \times 10^6 Hz} = 3.33 \times 10^{-8} s$$

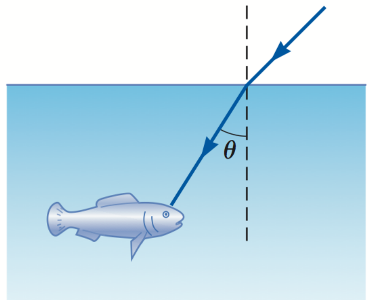
   (b) Find the maximum amplitude and direction of the magnetic field of the wave.

   Solution:
   $$B_{max} = \frac{E_{max}}{C} = \frac{150N/C}{(2.99 \times 10^8)\,\frac{\mathrm{m}}{\mathrm{s}}} = 5.02 \times 10^{-7} T$$
   Since motion is in the x direction, the E-filed is in the y direction, then the magnetic field must be in the z direction.

   (c) Find the average value of the intensity of the wave.

   Solution:
   $$I = S_{avg} = \frac{E_{max}^2}{2\mu_0 c} = 2.99 \times 10^1 W/m^2$$

7. $\boxed{\text{10 points}}$ Suppose you are a bird hunting for fish. You like a particular species of fish that has evolved to dive deeper when it spots you, so you want to be quick to capture it. If the fish is 0.30 m below the surface of water, and you are gliding just above the surface of the water, how close (in the horizontal direction, and in terms of total distance) can you get to the fish before it spots you? **Given:** $n_{air} = 1$, $n_{H_2O} = 1.33$



Solution: Snell's Law gives: $n_1 \sin\theta_1 = n_2 \sin\theta_2$, and the critical angle, the angle at which total internal reflection results, is found by setting $\theta_2 = 90$:

$$1.33 \sin\theta_c = 1.00 \sin(90) \;\rightarrow\; \sin\theta_c = \frac{1.00}{1.33} = 48.8°$$

You should see the fish when you were

$$\Delta x = 0.30m\,(\tan(48.8)) = 0.34m$$

and so the total distance is given by the Pythagorean theorem:

$$d = \sqrt{\Delta x^2 + \Delta y^2} = 0.46m$$

8. $\boxed{\text{10 points}}$ Suppose we have a thin film with index $n = 1.33$ and thickness $t = 113nm$. What maximum wavelength of light would result in constructive interference.

Solution:

$$2nt = \left(m + \frac{1}{2}\right)\lambda,\; m = 0, 1, 2, \ldots$$

We solve for λ,

$$\lambda = \frac{2nt}{\left(m + \frac{1}{2}\right)}$$

The maximum wavelength would correspond to $m = 0$, leaving:

$$\lambda = \frac{2nt}{\frac{1}{2}} = 4nt = 6.01 \times 10^{-7}m$$

9. $\boxed{\text{15 points}}$ When monochromatic ultraviolet light that has a wavelength equal to 400.0 nm is incident on a sample of an unknown metal, the emitted electrons have a maximum kinetic energy of 3.03 eV. **Given:** $hc = 1240\mathbf{eV} \cdot \mathbf{nm}$.

(a) What is the energy of an incident photon?

Solution: The energy of a photon is $E = hf$ or

$$E = \frac{hc}{\lambda} = \frac{1240\text{eV} \cdot \text{nm})}{\lambda} = \frac{1240}{400.0} eV = 3.10 eV$$

(b) What is the work function for potassium?

Solution: Recall that

$$KE_{max} = E_{photon} - \phi$$

where ϕ is the work function. Here, we have:

$$\phi = E_{photon} - KE = 3.10eV - 3.03eV = 0.070eV$$

(c) What would be the maximum kinetic energy of the electrons if the incident electromagnetic radiation had a wavelength of 600.0 nm?

Solution: Redo the above two equations and replace 400.0 nm with 600.0 nm to find:

$$KE = E_{photon} - \phi = 2.067eV - 0.070eV = 1.997eV$$

(d) What is the maximum wavelength of incident electromagnetic radiation that will result in the photelectric emission of electrons by a sample of potassium?

Solution: We take the limit for when the kinetic energy goes to zero and we have:

$$0 = E_{photon} - \phi \;\rightarrow\; \frac{hc}{\lambda} = \phi$$

In this case,

$$\frac{1240\text{eV} \cdot \text{nm}}{\lambda} = 0.070eV \;\rightarrow\; \lambda = 1.77 \times 10^4 \text{ nm}$$

10. $\boxed{\text{15 points}}$ (a) $\boxed{\text{2 points}}$ Find the rest energy of a proton in units of electron volts. Given: $m_p = 1.672 \times 10^{-27}$ kg, $c = (2.99 \times 10^8)\,\frac{\text{m}}{\text{s}}$, $1.00eV = 1.602 \times 10^{-19}J$.

Solution:

$$E_R = m_p c^2 = \left(1.672 \times 10^{-27}\ \text{kg}\right)\left(\left(2.99 \times 10^8\right)\frac{\text{m}}{\text{s}}\right)^2\left(\frac{1.00eV}{1.602 \times 10^{-19}J}\right) = 9.33 \times 10^8\text{eV}$$

(b) $\boxed{\text{2 points}}$ If the total energy of a proton is 5.0 its rest energy, what is the speed of the proton relative to the lab from which it is launched?

Solution:

$$E = 5.0 m_p c^2 = \gamma m_p c^2 \ \rightarrow\ \gamma = 5.0$$

And solving for v:

$$\frac{1}{\gamma^2} = 1 - \frac{v^2}{c^2} = \frac{1}{25.00}$$

Rearranging and solving for v:

$$v = 0.980c$$

(c) $\boxed{\text{2 points}}$ An event taking place in the proton's rest frame is clocked at 14.0 s, how long would that event seem to take in the lab frame? *Hint: The shortest time you can measure is in the rest frame*

Solution:

$$\Delta t' = \gamma \Delta t_p = \frac{14.0\text{s}}{\sqrt{1 - (0.980)^2}} = 70.00\ \text{s}$$

(d) $\boxed{\text{2 points}}$ Determine the kinetic energy of this proton in units of electron volts.

Solution:

$$K = E - m_p c^2 = (5.0 - 1)\, 9.33 \times 10^8\ \text{eV} = 3.73 \times 10^9\text{eV}$$

(e) $\boxed{\text{2 points}}$ What is the proton's momentum?

Solution: From the equation,

$$E^2 = p^2c^2 + (m_p c^2)^2 = \left(5.0 m_p c^2\right)^2$$

we can algebraically solve for p:

$$p = \sqrt{\frac{(5.0^2 - 1)\,(m_p c^2)^2}{c^2}} = 4.57 \times 10^9 \mathrm{eV/c}$$

Some possibly useful equations.

$$n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$$

$$I = S_{avg} = \frac{E_{max}^2}{2\mu_0 c}$$

$$B = \frac{E}{c}$$

$$c = \lambda f$$

$$c^2 = a^2 + b^2$$

$$T = \frac{1}{f}$$

$$\mu_0 = \left(4 \times \pi \times 10^{-7}\text{Tm/A}\right)$$

$$c = \left(2.99 \times 10^8\right) \frac{\text{m}}{\text{s}}$$

$$2nt = \left(m + \frac{1}{2}\right) \lambda, m = 0, 1, 2 \ldots$$

$$E = \frac{hc}{\lambda}$$

$$KE = E_{photon} - \phi$$

$$hc = 1240\text{eV} \cdot \text{nm}$$

$$m_p = 1.672 \times 10^{-27} \text{ kg}$$

$$q_p = 1.602 \times 10^{-19} \text{ C}$$

$$m_e c^2 = 0.511 \text{ MeV}$$

$$K = \gamma m c^2$$

$$K = E - mc^2$$

$$E^2 = p^2 c^2 + (m_p c^2)^2$$

$$x' = \gamma(x - vt)$$

$$t' = \gamma(t - \frac{vx}{c^2})$$

$$L' = \frac{L_p}{\gamma}$$

$$t' = \gamma t_p$$

End of exam