Name: _____

Multiple Choice

> Choose the **better** choice of all choices given.

1. Which of the following isn't a truth about quantum mechanics?

    A. Physicists are at a consensus about an interpretation of Quantum Mechanics.

    B. An electron can seem to interfere with itself when passing through double slits.

    C. Energy is quantized.

    D. Momentum is quantized

    E. A particle has a chance to be found in a region which should classically be impossible for it to be found in.

2. The square of the Schrödinger wave function is

    A. Equal to one.

    B. Not integrable.

    C. A probability density.

    D. Has no physical meaning.

    E. Is only physical at relativistic speeds.

3. Which of the following problem in physics was created by quantum mechanics?

    A. The particle/wave duality.

    B. The ultraviolet catastrophe of blackbody radiation

    C. The twin paradox

    D. The barn-door paradox

    E. The contradiction between the universal speed of light and Galilean transforms.

4. Suppose I have an atom that has 4 electrons with spin up and 3 electrons with spin down. If I'm able to ionize this atom by adding another electron, what spin will that electron be? *Hint: How many electrons can one have in each shell?*

    A. Spin up

    B. Spin down.

    C. Neutral spin.

    D. It is not possible to add another electron.

    E. You have to add two electrons, not one.

5. Which of the following experiments could never show quantum mechanics?

   A. Taking thousands of measurements and forming probabilistic models of those measurements.

   B. Taking thousands of identically prepared particles and measuring them one at a time

   C. Sending one electron at a time through a double-slit apparatus so that the electron can interfere with itself, and measuring the screen.

   D. Sending one electron at a time through a double-slit apparatus and measuring which slit it goes through, so that the electron won't interfere with itself.

6. If I know the position of a subatomic particle precisely, then

   A. I know nothing about the particle's momentum.

   B. I known a very limited amount about the particle's momentum.

   C. The particle must be at rest.

   D. The particle can't be at rest.

7. Einstein's term "spooky action at a distance" was referring to:

   A. The idea of entanglement, that two quantum particles could have connected natures no matter how far away they are from each other.

   B. The idea from relativity that two observers in different inertial frames could age differently.

   C. The idea from quantum mechanics that a particle could interfere with itself.

   D. The ghost that was throwing Einstein's dishes around his kitchen when he wasn't around.

8. In the Quantum Eraser Experiment, the interference pattern vanishes when...

   A. The detection screen is widened.

   B. The detection screen is shortened.

   C. The path the photon took is known.

   D. The path the photon took is unknown.

9. Low temperature superconductors occur due to:

   A. Electrons interacting with a lattice flow smoother

   B. A quantum effect where paired electrons act as bosons

   C. A quantum effect where paired electrons act as fermions

   D. Isolated electrons resist thermal kicks

10. According to the Dirac comb model, some materials are conductors and some materials are insulators due to:

     A. If the atoms are closer together, the thermal kicks are harder.

     B. The solution to the Schrodinger equation for the Dirac comb requires that some energy bands be empty due to the Heisenberg Uncertainty Principal.

     C. A classical effect in which thermal kicks are too effective for certain ranges of energy.

     D. $|\cos(\theta)| \leq 1$ restriction on the solution to Schrodinger's equation in the Dirac comb results in gaps in possible energy levels for electrons.

11. Why does doping of certain insulators turn them into semiconductors?

> **Solution:**
>
> Doping adds impurities to the material which results either in holes in filled energy bands or extra energy bands in the gap regions.

12. In the double slit experiment with electrons, where one electron at a time is shot at a double slit and then passes through and shows up on a screen, we can see an interference pattern in the distributions of where the electrons hit the screen. How can we cause this experiment to not show interference? That is, what can we change about the experiment to destroy the unique interference pattern?

> **Solution:** Put a detector in one or both of the slits to find the "which-way" information about which path the electron took.

Problems

13. Suppose a particle is confined to the x-axis from $x = 0$ to $x = 3$ in a quantum energy well. A quantum physicist measures the particle's position and write it down. She flushes out the particle so that the well is empty and then she puts in another particle with the identical set up as the first one, measures its position, writes it down, and so on. Eventually she gets enough data such that she can determine the probability density for finding the particle between $x = 0$ and $x = 3$, which is found to

$$P(x) = \frac{7x^6}{2187}$$

(a) What is the wave function for the position of this particle?

> **Solution:** Recall that $\psi^2(x) = P(x)$, so that $\psi(x) = \sqrt{P(x)}$. In this case,
>
> $$\psi(x) = \sqrt{\frac{7x^6}{2187}} = \frac{\sqrt{7}x^3}{\sqrt{2187}}$$

(b) What is the probability of finding the particle between $x = 2$ and $x = 3$?

> **Solution:**
> $$\int_2^3 \frac{7x^6}{2187}dx = \frac{x^7}{2187}\Big|_2^3 = 0.94$$

(c) What is the probability of finding the particle between $x = 0$ and $x = 2$?

> **Solution:**
> $$\int_0^2 \frac{7x^6}{2187}dx = \frac{x^7}{2187}\Big|_0^2 = 0.06$$

(d) Suppose that when the scientist measured the electron, her readings had an uncertainty of $\Delta x = 1.0 \times 10^{-9}m$. What is the smallest uncertainty that she could possibly have in her measurement for the particles velocity? **Given**: The mass of this particle is $3.5 \times 10^{-19}kg$ and $\hbar = 1.05 \times 10^{-34}m^2kg/s$

> **Solution:** Heisenberg's Uncertainty Principle states that
> $$\Delta x \Delta p \geq \frac{\hbar}{2}$$
> Where the best case scenario is to minimize the uncertainty (using $\Delta p = m\Delta v$):
> $$\Delta x \Delta p = \frac{\hbar}{2} \rightarrow \Delta v = \frac{\hbar}{2m\Delta x} = 1.50 \times 10^{-7}\frac{m}{s}$$

14. Suppose an electron in a Hydrogen atom is in the $n = 2$ state.

   (a) What must be the minimal energy of a photon to ionize this atom (that is, to liberate the electron)?

   > **Solution:**
   > $$E_{ion} = E_\infty - E_n = \frac{13.6 eV}{n^2} = 3.4 \text{ eV}$$

   (b) Now suppose that a photon with energy 4.4 eV hits the electron. The electron is liberated, how fast will it be going far from the hydrogen atom? Given: $m_e = 9.11 \times 10^{-31} kg$ and $1eV = 1.6 \times 10^{-19} J$. *Hint: If there is extra energy left over after liberation of the electron, that extra energy is converted into Kinetic Energy.*

   > **Solution:** Be sure to convert the energy to
   > $$E_{extra} = \frac{1}{2} m_e v^2 \;\rightarrow\; v = \sqrt{\frac{2 E_{extra}}{m_e}} = 5.99 \times 10^5 \frac{m}{s}$$

   (c) This electron flies off of the hydrogen atom and encounters an energy barrier of $3eV$. This is **more** than the kinetic energy of the electron, so classically, that electron should just bounce off and go back from whence it came. But in our interesting universe, there is a small chance that the electron will tunnel through the barrier! If the barrier has a width of 2.5×10^{-10} m, what is the probability that the electron will tunnel past the barrier? *Hint: The energy E of the electron is the kinetic energy you found above, and U is the potential barrier $3eV$.*

   > **Solution:** Recall that
   > $$C = \sqrt{\frac{2m(U - E)}{\hbar^2}}$$
   > and that the probability of tunneling is approximately
   > $$T \approx e^{-2CL}$$
   >
   > $$U - E = 2.0 \text{ eV} = 3.2 \times 10^{-19} \text{ J}$$
   > so that
   > $$2CL = 2\sqrt{\frac{2m(U - E)}{\hbar^2}} \left(2.5 \times 10^{-10} \text{ m}\right) = 3.62$$
   > and finally,
   > $$T = e^{-2CL} = 2.68 \times 10^{-2}$$

15. 5 points An HCL molecule vibrates with a frequency of 5.1×10^{13} Hz. Given $h = 6.626 \times 10^{-34}$ J · s.

(a) What is the smallest possible change of energy that this molecule could experience?

> **Solution:**
>
> $$\Delta E = hf = \left(6.626 \times 10^{-34} \text{J} \cdot \text{s}\right)\left(5.1 \times 10^{13} \text{ Hz}\right) = 3.38 \times 10^{-20} \text{ J}$$

(b) Suppose photons, each with energy 3.07×10^{-21} J, are used to excite this molecule to the next energy state. How many such photons would be needed?

> **Solution:** Divide ΔE for the molecule by the energy of each photon to find that it takes 11.0 photons.

(c) What is the frequency and wavelength of these photons?

> **Solution:** Recall that $E_p = hf$ and $c = \lambda f$.
>
> $$E_p = hf \;\rightarrow\; f = \frac{E_p}{h} = \frac{3.07 \times 10^{-21} \text{ J}}{6.626 \times 10^{-34} \text{J} \cdot \text{s}} = 4.64 \times 10^{12} \text{ Hz}$$
>
> And
> $$c = \lambda f \;\rightarrow\; \lambda = \frac{c}{f} = 6.45 \times 10^{-5} \text{ m}$$

16. (a) How much mass does a hydrogen atom gain or lose when it transitions from the $n = 5$ to the $n = 1$ state (in other words, Δm)? **Given** $c = 3.0 \times 10^8 m/s$ **and** $1eV = 1.6 \times 10^{-19} J$.

> **Solution:**
>
> $$\Delta m = \frac{\Delta E}{c^2} = \frac{-13.6 \text{ eV} \left(1 - \frac{1}{5^2}\right)}{c^2} = -2.32 \times 10^{-35} \text{kg}$$

(b) How much mass does a hydrogen gain or lose when it transitions from the $n = 5$ to the $n = 7$ state (in other words, Δm)? **Given** $c = 3.0 \times 10^8 m/s$ **and** $1eV = 1.6 \times 10^{-19} J$.

> **Solution:**
>
> $$\Delta m = \frac{\Delta E}{c^2} = \frac{-13.6 \text{ eV} \left(\frac{1}{7^2} - \frac{1}{5^2}\right)}{c^2} = 4.74 \times 10^{-37} \text{kg}$$

(c) Where does that gained or loss mass come from or go?

> **Solution:** Transformed from/to energy.

17. Scientist A measures a quantum system by injecting an electron into an infinite potential well, measuring its position with a laser beam. After each measurement of its position, Scientist A flushes the electron out and repeats the measurement thousands upon thousands of times.

(a) | 4 points | At what point in the experiment will the scientist find proof of quantum effects?

> **Solution:** When the scientist views the ensemble of thousands of results and plots a histogram of the locations she found the electrons, she will see evidence of a quantum probability wave rather than classical clustering.

(b) | 1 point | If the scientist were also to obtain some information about the momentum of the particle, what equation relates how accurately we can determine its momentum and position?

> **Solution:**
>
> $$\Delta x \Delta p \geq \frac{\hbar}{2}$$

(c) | 5 points | After doing this thousands of times, the scientist finds that the probability of locating the particle in the well is:

$$P(x) = \frac{1}{4}x^2 + x + 1$$

What is the quantum wave function for the electron in this well, $\psi(x)$? *Just a note, this wave function isn't physically realistic.*

> **Solution:**
> Since $P(x) = \psi^2(x)$ for simple functions, then
>
> $$\psi(x) = \sqrt{P(x)} = \sqrt{\frac{1}{4}x^2 + x + 1} = \sqrt{\left(\frac{1}{2}x + 1\right)^2} = \left(\frac{1}{2}x + 1\right)$$
>
> Again, it is important to note that this is a dummy function and isn't physically reasonable.

(d) | 5 points | Set up an equation to solve for the length of the well. You do not need to fully solve for L, just set up the equation. *Hint: Assume that the left side of the well starts at $x = 0$ and recall that the probability over the entire well has to equal 1.*

Solution:

$$\int_0^L \left(\frac{1}{4}x^2 + x + 1 \right) dx = \frac{1}{12}x^3 + \frac{x^2}{2} + x \Big|_0^L = \frac{L^3}{12} + \frac{L^2}{2} + L = 1$$

Equation Sheet

Some or all of these equations may be useful for the final exam.

$$E = nhf, n = 1, 2, \cdots$$

$$\Delta E = \Delta mc^2$$

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

$$E_n = \frac{-13.6 \text{ eV}}{n^2}$$

$$\Delta E_n = -13.6 \text{ eV} \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$$

$$\frac{-\hbar^2}{2m} \frac{d^2 \psi(x)}{dx^2} + U\psi(x) = E\psi(x)$$

$$\int_{-\infty}^{\infty} \psi^2(x) dx = 1$$

$$P(x) = \psi^2(x)$$

$$P(a \leq x \leq b) = \int_a^b \psi^2(x) dx$$

$$KE = \frac{1}{2}mv^2$$

$$C = \sqrt{\frac{2m(U - E)}{\hbar^2}}$$

$$T \approx e^{-2CL}$$

$$p = mv$$

$$\Delta p = m\Delta v$$

$$c = \lambda f$$