Physics 280 Quantum Mechanics Lecture

Spring 2015

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Objectives

- Review Early Quantum Mechanics

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- Schrödinger's Wave Equation

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- Heisenberg Uncertainty

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- Strange Contradictions

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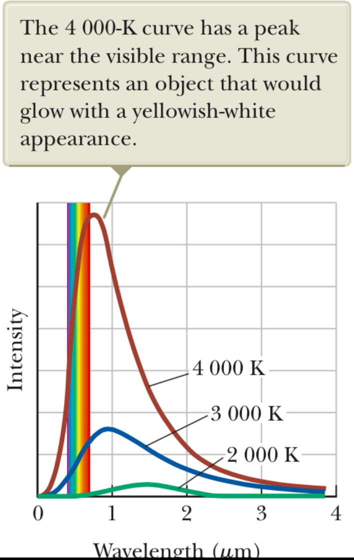
- Einstein takes this result at face value and shows that photons are quantized in the same way, explaining the photoelectric effect.
- Bohr adapts the idea of quantization to "explain" the Rydberg equation and why the emission/absorption spectrum is quantized. He quantizes angular momentum.

A new truth

Energy is quantized.

The Ultraviolet Catastrophe



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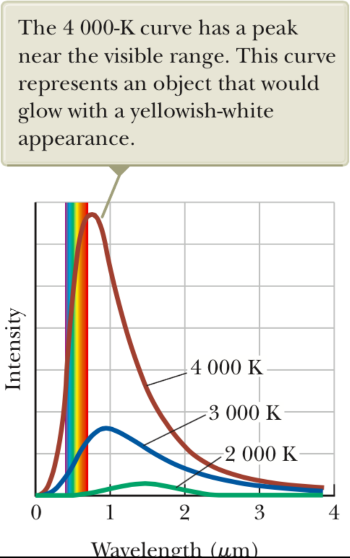

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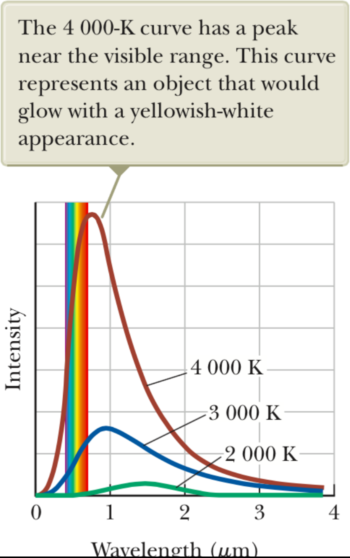

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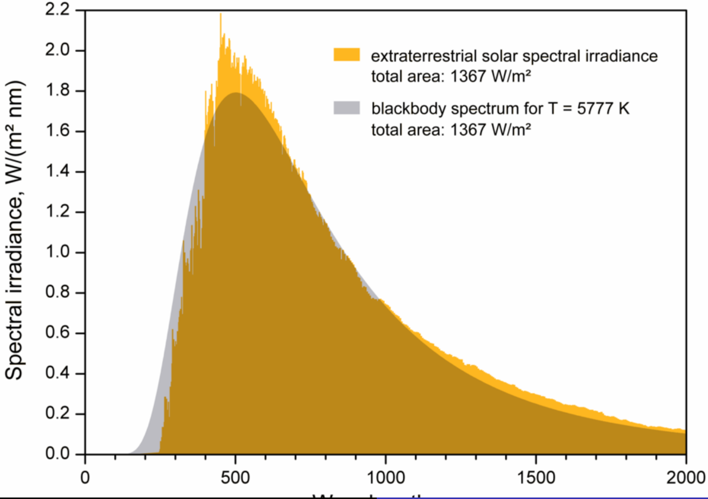
Wien's law can explain the shift of the peak leftwards with temperature.

$$\lambda_{max} T = \left(0.2898 \times 10^{-2}\right) \text{ mK}$$

Why we can approximate some glowing bodies as blackbody radiation



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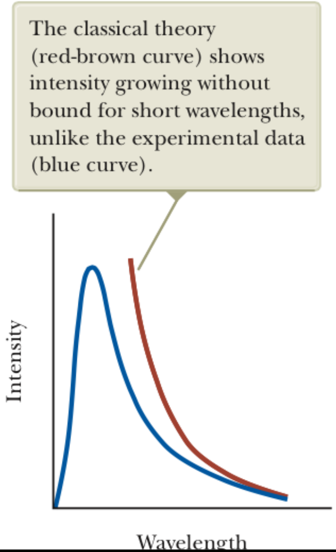

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$$P = \sigma A e T^4$$

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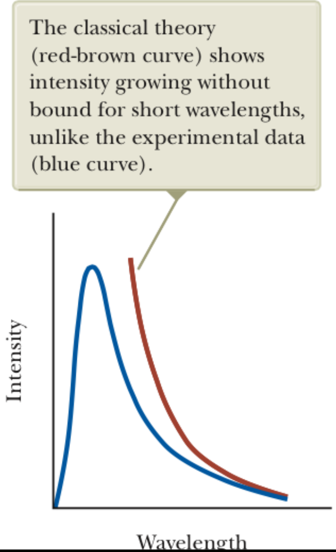

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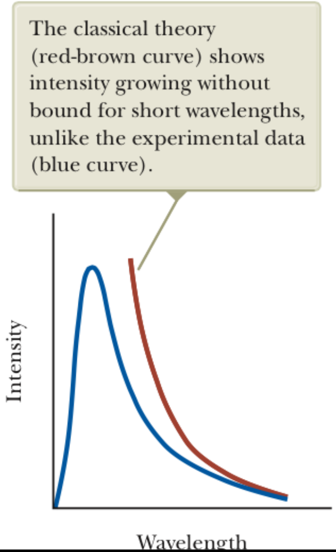

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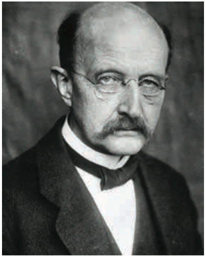
Classical physics could take us no further, and it was wrong.

The Ultraviolet Catastrophe: Resolved Reluctantly



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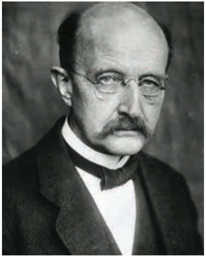

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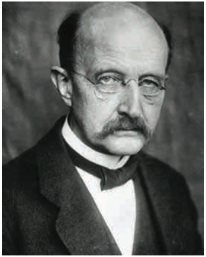

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$$I(\lambda, T) = \frac{2\pi hc^2}{\lambda^5 \left(e^{\frac{hc}{\lambda k_B T}} - 1 \right)}$$

Atomic mystery

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- Another mystery dominated atomic physics–nobody could explain the spectra of gasses.
- We might expect a continuous distribution of wavelengths, but instead we find discrete line spectrum called the *emission spectrum.*
- Passing white light through gasses result in discrete missing wave- lengths, this is called the absorption spectrum.

Atomic mystery

- Example emission (hydrogen, mercury, and neon) and absorption spectrum for hydrogen:


(a)


(b)


(c)

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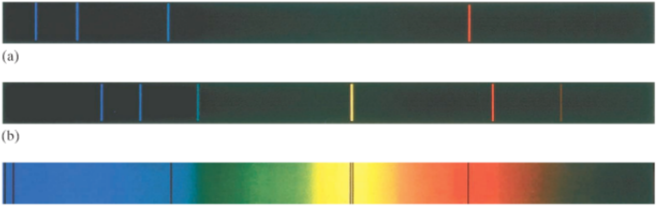

(a) (b) (c)

- 19th century physicists can't explain these spectra.

Atomic mystery

- Balmer found an empirical equation that correctly predicted the wavelengths of four of the visible emission lines; Rydberg expanded this equation to find all emission lines:

$$\frac{1}{\lambda} = R_H \left(\frac{1}{2^2} - \frac{1}{n^2} \right), \quad n = 3, 4, 5, \cdots$$

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- The shortest wavelength is found when $n \to \infty$ is called the series limit with wavelength 364.6 nm (ultraviolet).

Atomic mystery

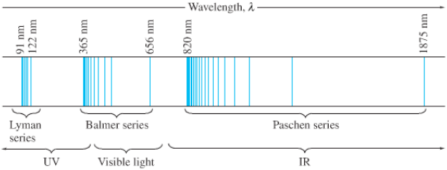
Other physicists started experimenting with these numbers and found similar equations that described other lines in the spectrum:

$$\begin{aligned}
\text{Lyman series: } \frac{1}{\lambda} &= R_H\left(1-\frac{1}{n^2}\right), \quad n = 2, 3, 4, \cdots \\
\text{Paschen series: } \frac{1}{\lambda} &= R_H\left(\frac{1}{3^2}-\frac{1}{n^2}\right), \quad n = 4, 5, 6, \cdots \\
\text{Bracket series: } \frac{1}{\lambda} &= R_H\left(\frac{1}{4^2}-\frac{1}{n^2}\right), \quad n = 5, 6, 7, \cdots
\end{aligned}$$

It all works very well, but nobody knows why.

Atomic mystery

FIGURE 27–23 Line spectrum of atomic hydrogen. Each series fits the formula $\frac{1}{\lambda} = R\left(\frac{1}{n'^2} - \frac{1}{n^2}\right)$ where $n' = 1$ for the Lyman series, $n' = 2$ for the Balmer series, $n' = 3$ for the Paschen series, and so on; n can take on all integer values from $n = n' + 1$ up to infinity. The only lines in the visible region of the electromagnetic spectrum are part of the Balmer series.



Quantum solution

A physicists named Niels Bohr saw that energy quantization had solved the blackbody and photoelectric problem, and he wondered if it could solve the mystery of the atomic spectrum lines as well. Here is a basic outline of his reasoning.

Quantum solution

- Assume the classical model of the electron orbiting the nucleus of the hydrogen atom under electrical forces. In this case the total energy is

$$E = K + U = \frac{1}{2} m_e v^2 - k_e \frac{e^2}{r}$$

and since we assume the electron is going in a circle, the electric force must act as a centripetal force:

$$\frac{k_e e^2}{r^2} = \frac{m_e v^2}{r} \implies v^2 = \frac{k_e e^2}{m_e r}$$

Using this value for velocity, we have kinetic energy:

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- So far, so classical.

Quantum solution

- Next comes the quantum step. Assume that only certain orbits are stable (called *stationary states*, which validates are previous assumption of using classical physics). The atom emits radiation when it makes a *quantum leap* from one state to another. The change in its energy after making this leap is *quantized*:

$$E_f - E_i = hf\,, \quad \text{positive value means absorbed, negative value means e}$$

Quantum solution

- Bohr next a new leap: quantizing the electron's orbital angular momentum:

$$m_e v r = n\frac{h}{2\pi} = n\hbar$$

where $\hbar = h/2\pi$. The energy quantization was enough to fluster some, but this latter assumption was a radically new expansion of the concept of quantization.

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- Now lets apply this radical concept:

$$v^2 = \frac{n^2\hbar^2}{m_e^2 r^2} = \frac{k_e e^2}{m_e r} \implies r_n = \frac{n^2\hbar^2}{m_e k_e e^2}, \quad n = 1, 2, 3$$

Quantum solution

- Call the orbit with the smallest radius the Bohr radius (when $n = 1$):
$$a_0 = \frac{\hbar^2}{m_e k_e e^2} = 0.0529nm$$
Plug this all back into the energy equation to find:
$$E_n = -\frac{k_e e^2}{2a_o}\left(\frac{1}{n^2}\right), \quad n = 1, 2, 3, \cdots$$
or numerically:
$$E_n = -\frac{13.606eV}{n^2}, \quad n = 1, 2, 3, \cdots$$
Now let's find out the frequency of light emitted from a quantum leap:
$$f = \frac{E_i - E_f}{h} = \frac{k_e e^2}{2a_0 h}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

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- That looks familiar...

$$\frac{1}{\lambda} = \frac{f}{c} = \frac{k_e e^2}{2a_0 hc}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

Plugging in all of our numbers:

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- We see it again: integers describing physical reality–one of the key signatures of quantum mechanics. This made most physicists uncomfortable, but also energized by the revolutionary spirit in the air.

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- Bohr proposes electron orbital angular momentum is quantized (1913), solves hydrogen spectrum problem, $E_n = \frac{-13.606\text{eV}}{n^2}$
- de Broglie (1923/24) proposes that if photons are waves and particles, so are massive particles like electrons:

$$p = mv = \frac{E}{c} = \frac{hf}{c} = \frac{h}{\lambda} \rightarrow \lambda = \frac{h}{mv}$$

Schrödinger's formalization

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- We seek a matter wave such that Ψ^2 is proportional the probability of finding an electron at a certain place and time.

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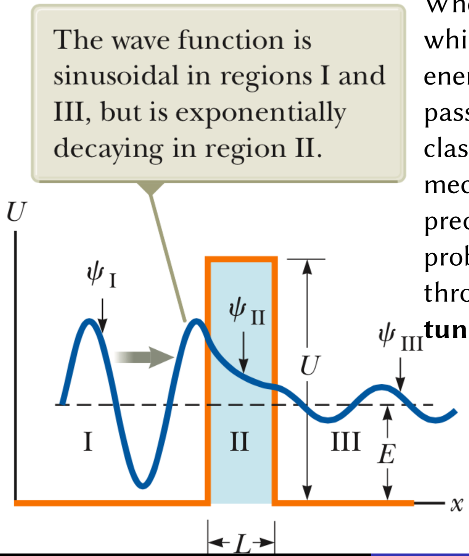
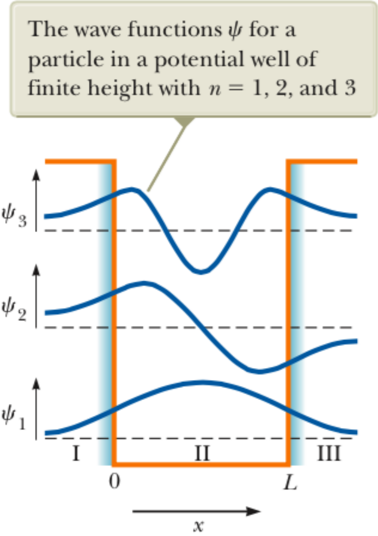
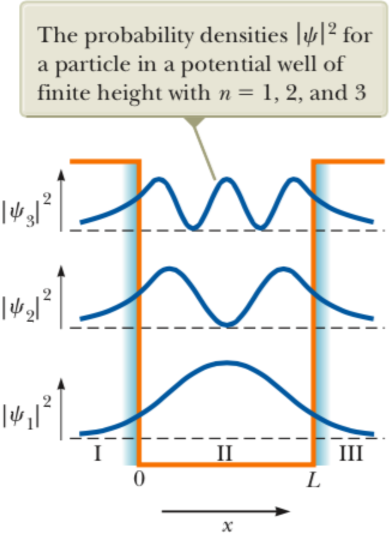
This is the **Heisenberg Uncertainty Principal.**

Probability



When there is a barrier which has too much energy for a particle to pass through in terms of classical physics, quantum mechanics correctly predicts that there is still a probability it will pass through. This is called **tunneling**.


The wave functions ψ for a particle in a potential well of finite height with n = 1, 2, and 3
ψ_3
ψ_2
ψ_1
I
II
III
0
L
x


The probability densities $|\psi|^2$ for a particle in a potential well of finite height with n = 1, 2, and 3
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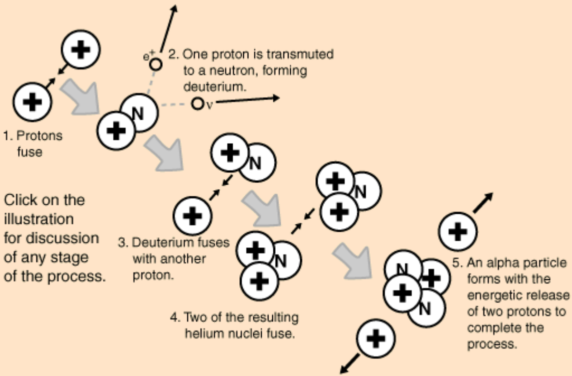
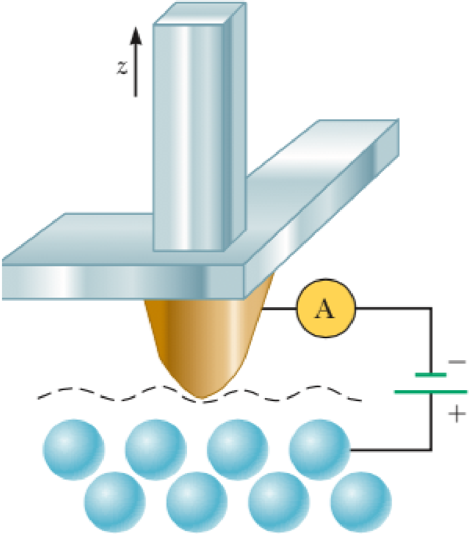
Proton-Proton Fusion

This is the nuclear fusion process which fuels the Sun and other stars which have core temperatures less than 15 million Kelvin. A reaction cycle yields about 25 MeV of energy.



Some details of the nuclear reactions involved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