1. Consider the interface of water with class (say at the bottom of a beaker filled with water). Given: The index of refraction for water is $n_w = 1.33$ and the index of refraction for this particular glass is $n_g = 1.52$. A ray of light with incident angle $\theta_w = 60.0°$ comes from water towards the glass (drawings are encouraged!). Find the angle of reflection and refraction.

Solution: The angle of reflection is the same as the incident angle: $\theta_w = \theta_r = 60.0°$.
Refraction can be set up with Snell's Law:

$$n_w \sin\theta_w = n_g \sin\theta_g$$

$$\sin\theta_g = \frac{n_w}{n_g}\sin(60) = \frac{1.33}{1.52}\sin(60) = 0.758 \ \rightarrow \theta = \sin^{-1}(0.758) = 49.3°$$

2. The wavelength of the red light from a helium-neon laser is 633 nm in air but only 474 nm in the aqueous humor inside of your eye. Calculate the index of refraction of this substance and the speed and frequency of this light in it. *Hint: The frequency of light doesn't change!*

Solution:
Recall that the speed of light in a vacuum (empty space) is c and in a medium with index $n > 1$ is $v = c/n$. Also recall that $c = \lambda f$, and inside the medium, $v = \lambda_n f_n$. We then have:

$$\lambda_n f_n = c/n = (\lambda/n)f$$

And so:

$$n = \frac{\lambda}{\lambda_n} = \frac{633}{474} = 1.34$$

This then gives a speed of:

$$v = \frac{c}{n} = \frac{\left(2.99\times10^8\right)\frac{\mathrm{m}}{\mathrm{s}}}{1.34} = 2.25\times10^8\frac{m}{s}$$

3. Sunlight reflects off the smooth surface of a swimming pool. For what angle of reflection is the reflect light completed polarized? Given: $n_{air} = 1.00, n_{H_2O} = 1.33$; $\sin(90-\theta) = \cos\theta$. Drawings are encouraged.

Solution: Recall that we have total internal polarization when the angle between the reflected ray is 90 degrees to the angle of the refracted ray. That is:

$$\theta_p + 90 + \theta_w = 180 \ \rightarrow \ \theta_w = 90 - \theta_p$$

We now apply Snell's law to get:

$$n_{air}\sin(\theta_p) = n_{H_2O}\sin(\theta_w) = n_{H_2O}\sin(90-\theta_p) = n_{H_2O}\cos(\theta_p)$$

Rearranging to get:

$$\frac{1.33}{1} = \tan(\theta_p) \ \rightarrow \ \theta_p = \tan^{-1}(\frac{1.33}{1}) = 53.1$$

4. While conducting a photo-electric effect experiment with light of a certain frequency, you find that a reverse potential difference of 1.25 V is required to reduce the current to zero. Find KE_{max}, the maximum Kinetic energy. Given, the charge of the electron: $e = 1.60 \times 10^{-19}C$. When the voltage is turned back down to zero, how fast will the fastest electrons travel? Given, the mass of an electron: $m_e = 9.11 \times 10^{-31}kg$.

Solution: Recall that when the current has come to a full stop, all electrons including the ones with maximum kinetic energy are held onto the plate:

$$KE_{max} = eV_0 = \left(1.60 \times 10^{-19}C\right)(1.25V) = 2.00 \times 10^{-19}J$$

Setting that equal to $(1/2)mv^2$ and solving for v:

$$v = \sqrt{\frac{2\left(2.00 \times 10^{-19}J\right)}{9.11 \times 10^{-31}kg}} = 6.63 \times 10^5 \frac{m}{s}$$

5. An interesting proposition. Imagine reversing the photoelectric effect. Imagine that the voltage between the plates is off when light strikes one plate and liberates an electron, but before the electron reaches the second plate, the voltage is increased to stopping potential so that the electron slows down, comes to a stop, and then accelerates back towards the plate, impacting the plate with the same kinetic energy it had at liberation.

   Surprise! A photon is released.

   Electrons in an x-ray tube accelerate through a potential difference of 10.0 kV before striking a metal target. If an electron produces one photon on impact with the target, what is the minimum wavelength of the resulting photon (which is an x-ray here)? Given, the charge of the electron: $e = 1.60 \times 10^{-19}C$ and $hc = 1240eV \cdot nm$ is Planck's constant times the speed of light given in eV units and nm units.

Solution:

Relating the maximum kinetic energy to the voltage difference to the photon energy:

$$KE_{max} = eV = hf = \frac{hc}{\lambda}$$

Plugging in,

$$10.0 \times 10^3 eV = \frac{1240eV \cdot nm}{\lambda}$$

$$\lambda = 0.124nm$$

6. A given doublet-slit arrangement had $d = (0.150e-3)\,\mathrm{m}$, $L = (1.40)\,\mathrm{m}$, $\lambda = (643e-9)\,\mathrm{m}$, and we consider the results of sending light through the arrangement that at a point P located $y = (1.80e-2)\,\mathrm{m}$ from the center of a screen (see Figure P37.19).

(a) What is the path difference δ for the rays from the two slits arriving at P.

> **Solution:** We use the standard geometrical equation,
>
> $$\delta = d\sin\theta \approx d\tan\theta = d\frac{y}{L} = (1.93e-06)\ \text{m}$$

(b) Express this path difference in terms of λ.

> **Solution:** We can express it as a ratio of λ:
>
> $$\delta = \left(\frac{\delta}{\lambda}\right)\lambda = (3.00e+00)\,\lambda$$

(c) Does this correspond to a minimum, maximum, or intermediate on the screen?

> **Solution:** Since it is an integer factor of the wavelength, it will be in sync and result in a maximum.

7. Consider a radio-wave transmitter and receiver separated by a distance of $d = 50$ m both of height $h = 35$ m. Determine the longest wavelengths that interfere constructively.

> **Solution:** By the law of reflection, the angle of the waves upon reflection are the same (compared to incidence relative to the normal vector). This means that the ray-trace shown reflecting in Figure P37.57 reflects off of the ground exactly halfway between the towers, and so the distance for the entire reflecting ray is $2\times\sqrt{h^2+(d/2)^2}$ (using the Pythagorean theorem).
>
> If we don't include the reflection phase-shift then the path difference is
>
> $$\delta = 2\times\sqrt{h^2+\left(\frac{d}{2}\right)^2} - d$$
>
> A phase shift of 180 degrees is equivalent to progressing $\lambda/2$ in wavelength, so
>
> $$\delta = 2\times\sqrt{h^2+\left(\frac{d}{2}\right)^2} + \frac{\lambda}{2} - d$$
>
> Our condition for the longest wavelength is $n=1$ so that $\delta=\lambda$ for constructive interference whereby,
>
> $$\begin{aligned} \lambda &= 2\times\sqrt{h^2+\left(\frac{d}{2}\right)^2} + \frac{\lambda}{2} - d \\ \frac{\lambda}{2} &= 2\times\sqrt{h^2+\left(\frac{d}{2}\right)^2} - d \\ \lambda &= 2\left(2\sqrt{h^2+\left(\frac{d}{2}\right)^2} - d\right) \\ &= (7.20e+01)\ \text{m} \end{aligned}$$

8. Sound with a frequency of 650 Hz from a distant sources (means assume that the wave is coherent and planar) passes through a door 1.10 m wide in a sound-absorbing wall. Find the number and angular direction of the diffraction minima at listening posts along a line parallel to the wall.

Solution: We use the diffraction minima equation $\sin\theta_{\text{dark}} = m\frac{\lambda}{a}$, $m = \pm 1, \pm 2, \pm 3, \cdots$. Also, we have the speed of sound as being $\left(340.29\frac{\text{m}}{\text{s}}\right)$ at sea level (on average). Sow we also know that the wavelength will be $\lambda = v_s/f = (5.24e-01)$ m.

Now that we have all that on paper, we need to keep in mind something quite important: $|\sin\theta| \leq 1$. So let's look at our particular equation.

$$\sin\theta_{\text{dark}} = m\frac{\lambda}{a} = m\,(4.76e-01)$$

If $m = 3$, then we have $\sin\theta_{\text{dark}} = (1.43e+00) > 1$, which is not possible, and if we have $m = 2$ $\sin\theta_{\text{dark}} = (9.52e-01) < 1$ so we have to have that $m = -2, -1, 1, 2$, so there are only four minima. So

m	$\theta = \sin^{-1}\left(m\frac{\lambda}{a}\right)$
-2	$(-7.21e+01)$
-1	$(-2.84e+01)$
1	$(2.84e+01)$
2	$(7.21e+01)$

9. A painting with dots of pure pigment with approximate diameter $(2e-3)$ m. Assume light of wavelength $\lambda = (500e-9)$ m and the average human pupil diameter is $(5e-3)$ m. How far away must the average person be to not see the pixels?

Solution: We use the equation

$$\theta_{min} = 1.22\frac{\lambda}{D}$$

We can use the small angle approximation here to set up a triangle whose one side is the distance from the painting and whose other side is the distance between the pixels, so that,

$$\theta_{min} \approx \tan\theta_{min} = \frac{d}{L} = 1.22\frac{\lambda}{D}$$

$$\frac{(2e-3)\ \text{m}}{L} = 1.22\frac{(500e-9)\ \text{m}}{(5e-3)\ \text{m}}$$

We solve for L to find,

$$L = (1.64e+01)\,m$$

10. The helium-neon laser with wavelength $\lambda = (632.8e-9)\,m$ is used to calibrate a diffraction grating. If the first-order maximum occurs at 20.5°, what is the spacing between adjacent grooves in the grating?

Solution: The equation for diffraction gratings is

$$d\sin\theta_{\text{bright}} = m\lambda, \quad m = 0, \pm 1, \pm 2, \pm 3, \cdots$$

We solve for d when $m = 1$,

$$d = \frac{\lambda}{\sin(20.5)} = (1.81e - 06)\, m$$

11. A diffraction grating has 4200 rulings/cm. On a screen 2.00 m from the grating, it is found that for a particular order m, the maxima corresponding to two closely spaced wavelengths of sodium (589.0 nm and 589.6 nm) are separated by 1.54 mm. Determine the value of m.

Solution: We begin with the diffraction equation,

$$d \sin\theta = m\lambda$$

With the small angle approximation, $\sin\theta \approx \tan\theta = y/L$

$$d\frac{y}{L} = m\lambda$$

Given the grating of 4200 rulings/cm, this corresponds to a grating distance of $(2.38e - 06)$ m.

The difference between the different wavelengths is,

$$\frac{d}{L}(y_2 - y_1) = m(\lambda_2 - \lambda_1)$$

We plug in everything we know:

$$\frac{\frac{(2.38e\text{−}6 \text{ m})}{2.00 \text{ m}}((1.54e - 3) \text{ m})}{(0.6 \text{ nm})} = m = 3$$

But this is wrong! Small angle approximation fails us here. (Solve for θ for $m = 1$ to see why). Instead we can use a trig-identity:

$$\sin\theta = \frac{\tan\theta}{\sqrt{1 + \tan^2\theta}}$$

After a lot of algebra, you get $m = 2$.

12. Light from a helium-neon laser $\lambda = 632.8$ nm is incident on a single slit. What is the maximum width of the slit for which no diffraction minima are observed?

Solution:

$$\sin\theta = m\frac{\lambda}{a}$$

requires that the magnitude of the right-side is less than or equal to one. Let $m = 1$ which is the best case scenario, so then the maximum width after which no maxima are observed would be $a = \lambda = 632.8$ nm.