Physics 280 Lecture 2

Summer 2016

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Objectives

- Review Lorentz Coordinate Transforms and principles of relativity

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- Be able to understand and perform Lorentz Velocity Transforms

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- Understand relativistic momentum
- Understand relativistic work and energy

Review: Galilean Relativity

"The laws of mechanics must be the same in all inertial frames of reference."

$$x' = x + ut$$

$$t' = t$$

Relativity

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- The principle of relativity: The laws of physics must be the same in all inertial reference frames.
- The constancy of the speed of light: The speed of light in vacuum has the same value, $(2.99 \times 10^{8})\,\frac{\mathrm{m}}{\mathrm{s}}$, in all inertial frames, regardless of the velocity of the observer or the velocity of the source emitting the light.

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KEY

The shortest amount of time for an event is measured in the proper-frame (the frame in which the event is at rest).

Length Contraction

The Lorentz Factor:

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Objects are measured with longest length in their rest-frame.

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Implication: Not only do we have time dilation, but time and space are tied together. Implication: The definition of simultaneous differs between observers. If something is simultaneous in one frame, it may not be so in another frame.

Lorentz Velocity Transformations

The Lorentz Transforms between frame S and frame S' where the relative speed between the frames is v is:

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These transforms are valid for infinitesimals as well:

$$dx' = \gamma (dx - udt)$$

$$dt' = \gamma \left(dt - \frac{udx}{c^2} \right)$$

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$$\frac{dx'}{dt'} = \frac{\frac{dx}{dt} - u}{1 - \frac{u}{c^2}\frac{dx}{dt}} \rightarrow \frac{dx'}{dt'} = \frac{v_x - u}{1 - \frac{uv_x}{c^2}}$$

In order to not confuse relative velocity with the velocity of an object measured in frame S, write the relative velocity as u and the velocity of a measured object as v_x.

Velocity transforms: example

Two spacecraft A and B are moving in opposite direction. An observer on the Earth measures the speed of spacecraft A to be 0.750c and the speed of spacecraft B to be 0.850c. Find the velocity of spacecraft B as observed by the crew on spacecraft A.

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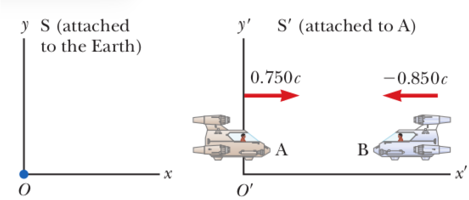

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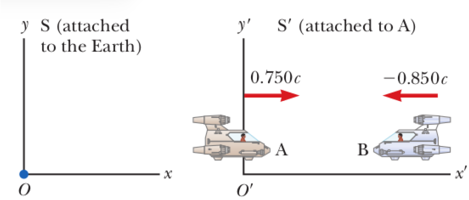

Identify $u = 0.750c$ and $v_x = -0.850c$ such that

$$v'_x = \frac{v_x - u}{1 - \frac{v_x u}{c^2}} = \frac{-0.850c - 0.750c}{1 - \frac{(-0.850c)(0.750c)}{c^2}} = -0.977c$$

Relativistic momentum and Newton's Second Law

The requirements of conservation of momentum require that our equation for momentum, $p = mv$ is modified:

$$\vec{p} = \frac{m\vec{v}}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma m\vec{v}$$

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This in turns modifies Newton's Second Law (F in direction of v):

$$F = \frac{d}{dt}\frac{mv}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{ma}{\left(\sqrt{1 - \frac{v^2}{c^2}}\right)^3}$$

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What happens when $v \rightarrow c$?

Relativistic momentum: example

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Classical:

$$p = mu = 9.11{\times}10^{-31} kg \left(0.750 \times 3.0 \times 10^{8} m/s\right) = 2.05{\times}10^{-22} kg{\cdot}m/s$$

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Relativity:

$$p = \frac{mu}{\sqrt{1 - \frac{u^2}{c^2}}} = 3.10 \times 10^{-22} kg \cdot m/s$$

Relativity and Energy

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Finally, the work energy theorem implies that

$$K = W = \int_{x_1}^{x_2} F dx \rightarrow [\text{Algebra and calc}] \rightarrow (\gamma - 1)\, mc^2$$

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What is the energy if the particle is at rest? With some algebra (see textbook) we get an equation for energy:

$$E^2 = \left(mc^2\right)^2 + (pc)^2$$

when $p = 0$:

$$E = mc^2$$

and when $m = 0$ (e.g. photons):

$$E = pc$$

Relativistic energy: part 1

Physicists tend to find that it is easier to express energy in terms of eV, the energy it takes to bring an electron across a potential of 1 Volt. Find the rest energy of a proton in units of electron volts.
Given: $1eV = 1.602 \times 10^{-19} J$, $m_p = 1.673 \times 10^{-27} kg$.

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$$E_p = 1.504 \times 10^{-10} J \left(\frac{1.00eV}{1.602 \times 10^{-19} J}\right) = 938 MeV$$

If the total energy of a proton is three times its rest energy, what is the speed of the proton?

Relativistic energy: part 2

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Solve for u:

$$9 = \frac{1}{1 - \frac{u^2}{c^2}} \rightarrow u = 0.943c$$

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$$p^2c^2 = 8\left(m_pc^2\right)^2$$

$$p = \sqrt{8}\frac{m_pc^2}{c} = \sqrt{8}\left(938MeV/c\right) = 2.65 \times 10^3 MeV/c$$