

# Physics 280 Lecture One

Summer 2016

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# Objectives

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- Be able to recognize the principles of relativity
- Be able to understand the radical truth of relativity
- Be able to apply the Lorentz transforms
- Be able to predict what a box will look like at relativistic speeds



- Electron acceleration



# Early evidence

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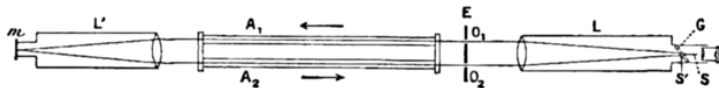
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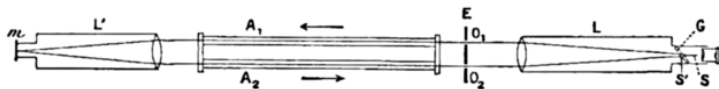
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- The Experiments of Rayleigh and Brace (1902, 1904)

# Fizeau experiment



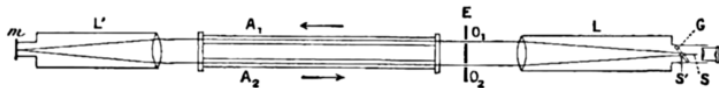
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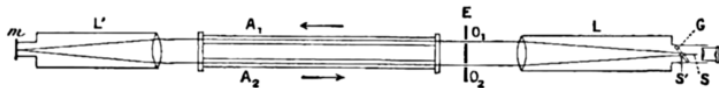
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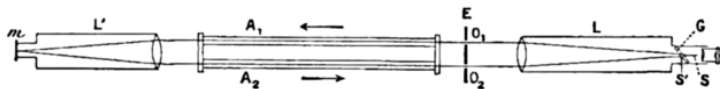
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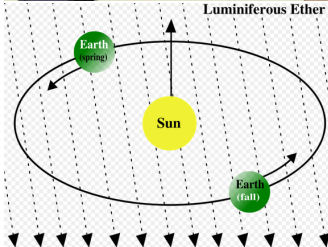
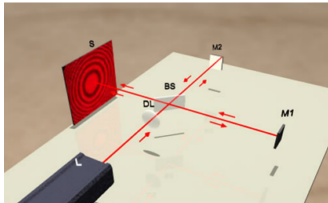


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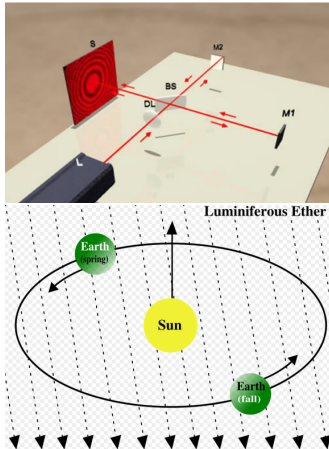
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- After reflecting at  $m$ , traveling back and recombining at  $S$ , interference patterns will reveal the speed difference between the two pipes.
- Actual finding:  $w_{+} = \frac{c}{n} + v \left(1 - \frac{1}{n^2}\right)$

# Michelson-Morley experiment



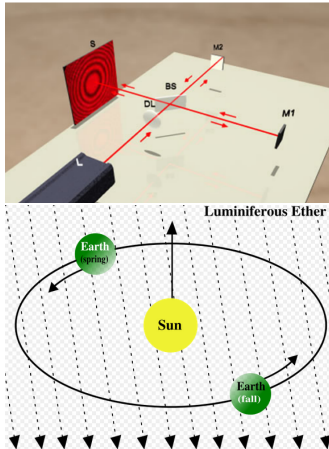
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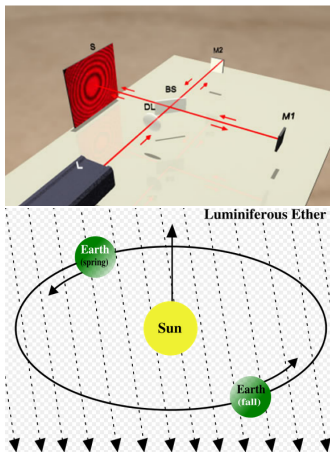
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- Earth is in motion through space, so there should be change in direction of motion
- A rotating interference apparatus sensitive enough to measure this speed difference failed to find any fringe shifts.
- “the most famous failed experiment in history”

# Michelson-Morley experiment

“there is no such thing as a ”specially favoured” (unique) co-ordinate system to occasion the introduction of the ether-idea, and hence there can be no ether-drift, nor any experiment with which to demonstrate it.” – Einstein

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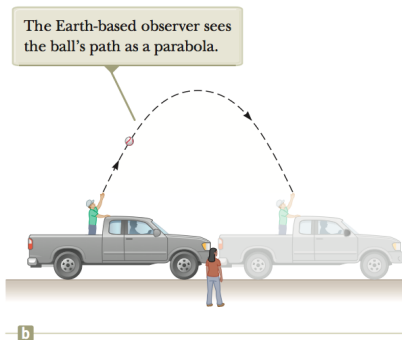
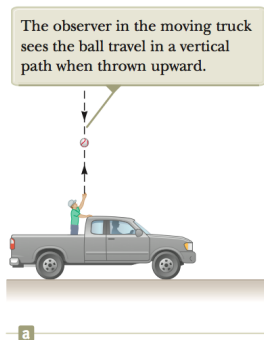
- The failure of these experiments to support the classical perspective, along with the predictions of uniform light-speed coming from EM theory, left physics with a major theoretical problem.
- We will learn about some other, even bigger problems later.
- Galilean Relativity must be wrong.

“The laws of mechanics must be the same in all inertial frames of reference.”

$$x' = x + vt$$

$$t' = t$$

# Galilean Relativity



Gregor throws a ball up at 5 m/s in a truck that is moving 20 m/s relative to Mei who is standing on the ground. How fast is the ball going according to Mei when it is first launched?

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$$x' = x + v_x t \rightarrow \frac{dx'}{dt} = \frac{dx}{dt} + v_x = 0 + 20\text{m/s} = 20\text{m/s}$$

$$y' = y + v_y t \rightarrow \frac{dy'}{dt} = \frac{dy}{dt} + v_y = 5\text{m/s} + 0 = 5\text{m/s}$$

So according to Mei,

$$\vec{V} = 20\text{m/s}\hat{\mathbf{i}} + 5\text{m/s}\hat{\mathbf{j}}$$

whereas according to Gregor,

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Both are right according to Galilean relativity, but the truth is much more interesting.

## ON THE ELECTRODYNAMICS OF MOVING BODIES

By A. Einstein  
June 30, 1905

It is known that Maxwell's electrodynamics—as usually understood at the present time—when applied to moving bodies, leads to asymmetries which do not appear to be inherent in the phenomena. Take for example the reciprocal electrodynamic action of a magnet and a conductor. The observable phenomenon here depends only on the relative motion of the conductor and the magnet, whereas the customary view draws a sharp distinction between the two cases in which either the one or the other of these bodies is in motion. For if the magnet is in motion and the conductor at rest, there arises in the neighborhood of the magnet an electric field with a certain definite energy, producing a current at the places where parts of the conductors are situated. But if the magnet is stationary and the conductor in motion, no electric field arises in the neighborhood of the magnet. In the conductor, however, we find an electromotive force, in which as well there is an accompanying energy, but which gives rise—according to our present notions to the two cases—directed to electric currents of the same path and intensity as those produced by the electric forces in the former case.

Examples of this sort, together with the unsuccessful attempts to discover any motion of the earth relatively to the "light medium," suggest that the phenomena of electrodynamics as well as of mechanics possess no properties corresponding to the idea of absolute rest. They suggest rather that, as has already been shown to the first order of small quantities, the same laws of electrodynamics and optics will be valid for all frames of reference for which the equations of mechanics hold good.<sup>1</sup> We will raise this conjecture (the purport of which will hereafter be called the "Principle of Relativity") to the status of a postulate, and also introduce another postulate, which is only apparently incompatible with the former, namely, that light is always propagated in empty space with a definite velocity  $c$  which is independent of the state of motion of the emitting body. These two postulates suffice for the attainment of a simple and consistent theory of the electrodynamics of moving bodies based on Maxwell's theory for stationary bodies. The introduction of a "luminiferous ether" will prove to be superfluous inasmuch as the view here to be developed will not require an "absolutely stationary space" provided with special properties, nor assign a velocity-vector to a point of the empty space in which electromagnetic processes take place.

The theory to be developed is based like all electrodynamics on the kinematics of the rigid body, since the assistance of any such theory here to do with the relationships between rigid bodies (systems of co-ordinates), clocks, and electromagnetic processes, necessitates consideration of the circumstances here at the root of the difficulties which the electrodynamics of moving bodies at present encounters.

### I. KINEMATICAL PART

#### § 1. Definition of Simultaneity

Let us take a system of co-ordinates in which the equations of Newtonian mechanics hold good.<sup>2</sup> In order to enable our presentation more precise and to distinguish this system of co-ordinates really from others which will be introduced hereafter, we call it the "stationary system."

If a material point is at rest relatively to this system of co-ordinates, its position can be defined relatively thereto by the employment of rigid standards of measurement and the methods of Euclidean geometry, and can be expressed in Cartesian co-ordinates.

If we wish to describe the motion of a material point, we give the values of its co-ordinates as functions of the time. Now we must here

- The principle of relativity: The laws of physics must be the same in all inertial reference frames.

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Examples of this sort, together with the unsuccessful attempts to discover any motion of the earth relatively to the "light medium," suggest that the phenomena of electrodynamics as well as of mechanics possess no properties corresponding to the idea of absolute rest. They suggest rather that, as has already been done in the first volume of my paper on "Electrodynamics and Optics" and will be valid for all frames of reference (or which the equations of mechanics hold good). We will raise this question the purpose of which will hereafter be called the "Principle of Relativity") to the status of a postulate, and also introduce another postulate, which is only apparently incompatible with the former, namely, that light is always propagated in empty space with a definite velocity  $c$  which is independent of the state of motion of the emitting body. These two postulates suffice for the attainment of a simple and consistent theory of the electrodynamics of moving bodies based on Maxwell's theory for stationary bodies. The introduction of a "luminiferous ether" will prove to be superfluous inasmuch as the view here to be developed will not require an "absolutely stationary space" provided with special properties, nor assign a velocity—relative to a point of the empty space in which electromagnetic processes take place.

The theory to be developed is based—like all electrodynamics—on the kinematics of the rigid body, since the assurance of size such theory have to do with the relationships between rigid bodies (systems of co-ordinates), clocks, and electromagnetic processes. Inevitable consideration of the consequences lies at the root of the difficulties which the electrodynamics of moving bodies at present encounters.

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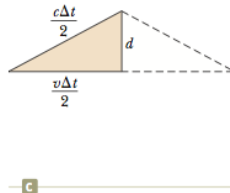
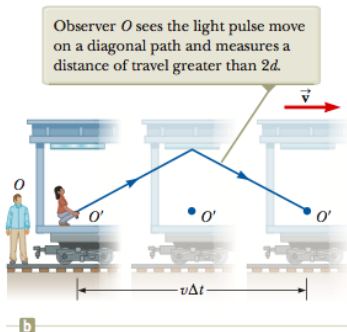
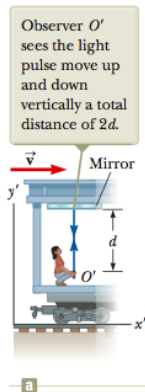
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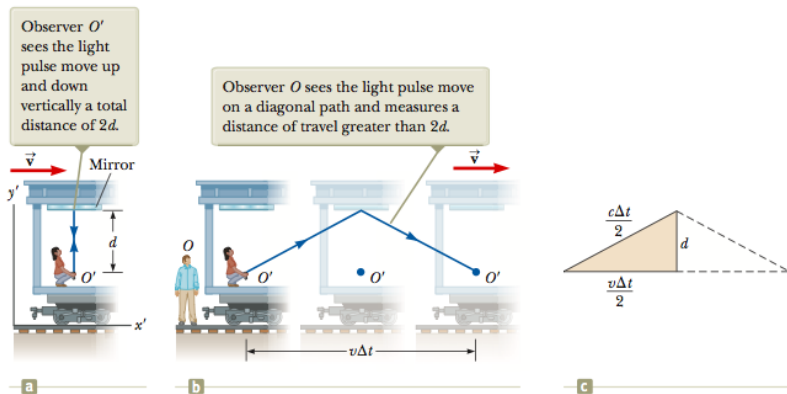
- The principle of relativity: The laws of physics must be the same in all inertial reference frames.
- The constancy of the speed of light: The speed of light in vacuum has the same value,  $(2.99 \times 10^8) \frac{m}{s}$ , in all inertial frames, regardless of the velocity of the observer or the velocity of the source emitting the light.

# Simple statement, Radical consequences





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Both observers must absolutely measure the speed of light to be the same,  $c = (2.99 \times 10^8) \frac{\text{m}}{\text{s}}$ .

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Who is right?

# Welcome to reality.

They are both right!

To distinguish their times, let us call the time measurement in the frame in which the event happens and is at rest the proper time (the clock here is at rest relative to the event) denoted  $t_p$ . Then relating the previous findings:

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WTF

Time is relative!

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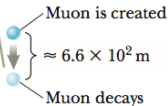
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Proof: Muons.

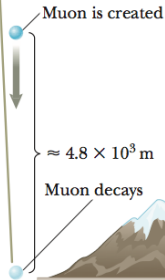
# Muon decays

Without relativistic considerations, according to an observer on the Earth, muons created in the atmosphere and traveling downward with a speed close to  $c$  travel only about  $6.6 \times 10^2$  m before decaying with an average lifetime of  $2.2 \mu\text{s}$ . Therefore, very few muons would reach the surface of the Earth.



a

With relativistic considerations, the muon's lifetime is dilated according to an observer on the Earth. Hence, according to this observer, the muon can travel about  $4.8 \times 10^3$  m before decaying. The result is many of them arriving at the surface.



b

# Muon decays

What will be the mean lifetime of a muon as measured in the laboratory if it is traveling at  $v = 0.60c$  with respect to the laboratory? Its mean lifetime at rest is  $2.20 \mu\text{s}$ . How far does it travel in the lab before decaying?

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Thus it will travel a distance:

$$d = v\Delta t = 0.60 \left( (2.99 \times 10^8) \frac{\text{m}}{\text{s}} \right) (2.8 \times 10^{-6} \text{s}) = 500 \text{m}$$



# GPS!

GPS satellites move at about 4000 m/s. Show that a good GPS receiver needs to correct for time dilation if it produces results consistent with atomic clocks accurate to 1 part in  $10^{13}$ . Hint, you will need the binomial expansion:  $1/\sqrt{1-x} \approx 1 + \frac{1}{2}x$  when  $x \ll 1$ .

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That may seem small, but is actually large compared to the accuracy of the atomic clock. The inaccuracies pile up and eventually the signal would be way off.

## One more thing...

Recall back to what I said in class about the flashlight on the truck. Imagine somebody in a truck driving at 10 m/s shines a flashlight in the direction of her motion. You are standing on the road as they pass by.

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**You both must measure the speed of light to be  $c$ .**



# Length Contraction

So let's consider one photon, sort of like we did with the baseball in the Galilean relativity example.

The position of that photon in the frame of a bystander on the ground should relate in some way to the position as measured by the person in the truck (whose speed is  $v$ ):

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Multiplying the previous two equations together gives

$$c^2 = \alpha^2 (c^2 - v^2) \rightarrow \alpha = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

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Objects are measured with longest length in their rest-frame.

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Implication: Not only do we have time dialation, but time and space are tied together. Implication: The definition of simultaneous differs between observers. If something is simultaneous in one frame, it may not be so in another frame.

# In-Class practice

An astronaut takes a trip to Sirius, which is located a distance of 8 light-years from the Earth. The astronaut measures the time of the one-way journey to be 6 years. If the spaceship moves at a constant speed of  $0.8c$ , how can the 8-ly distance be reconciled with the 6-year trip time measured by the astronaut?



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In the astronaut's frame,

$$\Delta t = L/v = 5ly/0.8c = 5ly/(0.8ly/yr) = 6yr$$