Name: ________________________________

Multiple Choice

Choose the better choice of all choices given.

1. Why can we only fit two electrons, one spin-up and the other spin-down, in an electron shell?
   - A. Because electrons are Bosons and can co-exist in any quantum state
   - B. Because electrons are Fermions and can co-exist in any quantum state
   - C. Because electrons are Bosons and can't co-exist in the exact same quantum state
   - D. Because electrons are Fermions and can't co-exist in the exact same quantum state
   - E. Because electrons are Bosons and can co-exist in any quantum state with the same spin
   - F. Because electrons are Fermions and can co-exist in any quantum state with the same spin
   - G. Because electrons are Bosons and can't co-exist in the exact same quantum state with the same spin
   - **H. Because electrons are Fermions and can't co-exist in the exact same quantum state with the same spin**
2. The integration of the square of the Schrödinger wave function over all possible positions is **never** equal to:
   - A. 1
   - **B. The probability of finding a particle in an infinitesimal region, e.g.** dx
   - C. The probability of finding an electron in a well
   - D. The probability of finding an electron that exists
3. In general, how are quantum effects measured in the lab?
   - **A. Statistically, but taking thousands of measurements and forming probabilistic models of those measurements.**
   - B. The spaces between energy levels is too small to see with our senses, so we aren't able to measure quantum effects.
   - C. Electrons are accelerated to near the speed of light in order to see the effects.
   - D. We connect our measurement devices up to a box with a cat in it.

4. What quantum feature of the electron enables us to put two electrons in each Hydrogen's orbital states?

Solution: Spin

5. If I know the position of a subatomic particle precisely, then
   A. I know nothing about the particle's position.
   B. I know nothing about the particle's energy.
   C. I know nothing about the particles momentum.
   D. I known a very limited amount about the particle's velocity.
   E. The particle must be at rest.
   F. The particle can't be at rest.

6. Proper length is
   A. Found in the frame at rest relative to the Universe.
   B. Found in the rest-frame of the object or distance between events/objects that is being measured.
   C. Found in the frame in which the time of an incident would be the longest.
   D. Found in the frame in which the object or distance between events/objects being measured is in motion.
   E. I like apples.

7. What are the two postulates of relativity?

Solution:

1. There is no preferred reference frame; the laws of physics are the same in all frames.
2. It is a law of physics that the speed of light is $c = (2.99 \times 10^8)\,\frac{\mathrm{m}}{\mathrm{s}}$.

8. Roughly speaking, Superconductivity is due to
   A. Electrons entering a hybrid state and acting like fermions
   B. Electrons entering a hybrid state and acting like bosons
   C. Electrons filling each energy level in pairs
   D. Electrons coupling with protons

9. Which of the following problem in physics wasn't fixed by quantum mechanics?
   A. The ultraviolet catastrophe of blackbody radiation
   B. The photoelectric mystery
   C. The helium spectrum mystery
   D. The constancy of the speed of light.

10. Which of the following problem in physics was created by quantum mechanics?
    A. The particle/wave duality.
    B. The ultraviolet catastrophe of blackbody radiation
    C. The twin paradox
    D. The barn-door paradox
    E. The contradiction between the universal speed of light and Galilean transforms.

11. Suppose I have an atom that has 5 electrons with spin down and 4 electrons with spin up. If I'm able to ionize this atom by adding another electron, what spin will that electron be? *Hint: How many electrons can one have in each shell?*
    A. Spin up
    B. Spin down.
    C. Neutral spin.
    D. It is not possible to add another electron.
    E. You have to add two electrons, not one.

12. Which of the following experiments could not show a quantum mechanics effect?
    A. Sending one electron at a time through a double-slit apparatus and measuring where it hits on a screen beyond the slits.
    B. Sending one electron at a time through a double-slit apparatus and measuring which slit it goes through and then measuring where it hits on a screen beyond the slits.
    C. Taking thousands of measurements of an isolated subatomic system that is reset after each measurement and forming probabilistic models of those measurements.
    D. Taking thousands of identically prepared particles and measuring them one at a time in identical conditions.

13. In your opinon, what is the most interesting quantum mechanical effect that you learned about in class?

Problems

14. Consider the Bohr model of the Hydrogen Atom.

(a) What observation about Hydrogen inspired Bohr to develop his model? *Hint: What observation of Hydrogen did not fit the classical expectation?*

Solution: The discreet spectrum

(b) How much energy is needed to ionize a hydrogen atom in the $n = 4$ state?

Solution:

$$E_{ion} = 0 - E_n = \frac{13.6eV}{n^2} = 8.5 \times 10^{-1} \text{ eV}$$

To get the frequency, use the equation for photon energy: $E = hf$. The change in energy in the previous part must come from that photon so that:

$$E = hf \;\rightarrow\; f = \frac{E}{h}$$

First, however, one must convert from eV to Joules as anything with Planck's constant that has to do with energy should be in Joules:

$$8.5 \times 10^{-1} \text{ eV} \left(\frac{1.6 \times 10^{-19} J}{1eV}\right) = 1.4 \times 10^{-19} \text{ J}$$

(c) What frequency photon would that require? **Given:** $1eV = 1.6 \times 10^{-19} J$
Then:

Solution:

$$f = \frac{E}{h} = \frac{1.4 \times 10^{-19} \text{ J}}{6.626 \times 10^{-34} \text{J} \cdot \text{s}} = 2.1 \times 10^{14} \text{ Hz}$$

(d) Now suppose that a photon with energy 1.1 eV hits the electron. The electron is liberated, how fast will it be going far from the hydrogen atom? Given: $m_e = 9.11 \times 10^{-31} kg$ and $1eV = 1.6 \times 10^{-19} J$. *Hint: If there is extra energy left over after liberation of the electron, that extra energy is converted into Kinetic Energy.*

> **Solution:** Be sure to convert the energy to
>
> $$E_{extra} = \frac{1}{2} m_e v^2 \;\rightarrow\; v = \sqrt{\frac{2E_{extra}}{m_e}} = 2.99 \times 10^5 \frac{m}{s}$$

(e) This electron flies off of the hydrogen atom and encounters an energy barrier of $U = 3eV$. This is **more** than the kinetic energy of the electron, so classically, that electron should just bounce off and go back from whence it came. But in our interesting universe, there is a small chance that the electron will tunnel through the barrier! Find what $U - E$ is, where E is the kinetic energy in electronvolts ev that you found above (its the extra left over energy).

> **Solution:**
>
> $$U - E = 2.7 \text{ eV} = 4.4 \times 10^{-19} \text{ J}$$

(f) Recall that

$$C = \sqrt{\frac{2m(U-E)}{\hbar^2}}$$

If $\hbar = 1.055 \times 10^{-34} \frac{m^2 kg}{s}$, m_e is the mass of the electron given above, and the barrier has a width of $L = 2.5 \times 10^{-10}$ m, find $2CL$.

> **Solution:**
>
> $$2CL = 2\sqrt{\frac{2m(U-E)}{\hbar^2}}\left(2.5 \times 10^{-10} \text{ m}\right) = 3.62$$

(g) What is the probability that the electron will tunnel through the barrier?

> **Solution:** Recall that
>
> $$T \approx e^{-2CL}$$
>
> so that
>
> $$T = e^{-2CL} = 2.68 \times 10^{-2}$$

15. Help the esteemed physicist Mace Windu find the work function of a mysterious metal. Dr. Windu has two light sources, one with 75.0 nm light and another with 125.0 nm light.

(a) What is the energy of a photon of the 75.0 nm light (solve for this in eV, not Joules)?

Solution:

$$E = \frac{hc}{\lambda} = 1.65 \times 10^1 \text{ eV}$$

(b) What is the energy of a photon of the 125.0 nm light (in eV)?

Solution:

$$E = \frac{hc}{\lambda} = 9.92 \text{ eV}$$

(c) Dr. Windu sets up his voltage plates so that the positive plate is the material from which the electrons are leaving and the negative plate is the plate to which they are heading. As you know, this will slow the electrons down, and if he turns the voltage up high enough, it will slow them to a complete stop and we won't have emitted electrons. In such a case, we have $KE = e \cdot \Delta V$.

Dr. Windu finds that it takes 6.52 V to terminate the flow of electrons from the plate being illuminated with his 125.0 nm light. What is the work function ϕ of the mysterious material?

Solution:

$$KE = E_{photon} - \phi \;\rightarrow\; \phi = E_{photon} - KE = 3.40 \text{ eV}$$

(d) How much voltage across the plates must Dr. Windu apply in order to stop the emission of electrons for the case where he is illuminating the material with 75.0 nm photons?

Solution:

$$KE = E_{photon} - \phi = 1.31 \times 10^1 \text{eV}$$

so the answer is,

$$\Delta V = 1.31 \times 10^1 \text{V}$$

since the charge on electrons is e anyway.

16. Suppose a particle is confined to the x-axis from $x = 0$ to $x = 3$, and that its wave function is given as $\psi(x) = Ax$.

    (a) What is A? *Note, this isn't a realistic function, just one that is solvable without tables of integrals.*

    Solution: We will use the **normalization** condition to solve for this:

    $$\int_{-\infty}^{\infty} \psi(x)^2 = 1$$

    $$\int_0^3 \psi^2(x)dx = \int_0^3 A^2x^2dx = \frac{A^2}{3}x^3 \mid_0^3 = 9A^2 = 1$$

    Thus $A = 1/3$ and the full wave function is:

    $$\psi(x) = \frac{x}{3}$$

    (b) Using the function from the previous part, find the probability of finding the particle between $x = 1$ and $x = 2$.

    Solution:

    $$P(x \in (1,2)) = \int_1^2 \psi^2(x)dx = \int_1^2 \frac{x^2}{9}dx = \frac{x^3}{27} \mid_1^2 = \frac{7}{27} = 0.26$$

    (c) Find the average position of the particle, which is defined as

    $$\int x\psi^2(x)dx$$

    This is called the expectation value of the position of the particle.

Equation Sheet

Some or all of these equations may be useful for the final exam.

$$E = nhf, n = 1, 2, \cdots$$

$$\Delta E = \Delta mc^2$$

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

$$E_n = \frac{-13.6 \text{ eV}}{n^2}$$

$$\Delta E_n = -13.6 \text{ eV} \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$$

$$\frac{-\hbar^2}{2m} \frac{d^2\psi(x)}{dx^2} + U\psi(x) = E\psi(x)$$

$$\int_{-\infty}^{\infty} \psi^2(x) dx = 1$$

$$P(x) = \psi^2(x)$$

$$P(a \leq x \leq b) = \int_a^b \psi^2(x) dx$$

$$KE = \frac{1}{2} mv^2$$

$$C = \sqrt{\frac{2m(U - E)}{\hbar^2}}$$

$$T \approx e^{-2CL}$$

$$p = mv$$

$$\Delta p = m\Delta v$$

$$c = \lambda f$$