Name: ________________________________

Multiple Choice

Choose the better choice of all choices given.

1. 2 points How many electrons can fit in one electron orbital shell?
   A. Only one
   B. Only two
   C. Only three
   D. Unlimited.

2. 2 points Which of the following isn't a truth about quantum mechanics?
   A. Energy is quantized
   B. Particles have wavelike properties.
   C. A cat can be dead and alive at the same time.
   D. A particle can be in a combination of two different states.

3. 2 points Given what we've learned about relativity, which of the following is better to say than "matter can neither be created nor destroyed":
   A. Matter can be converted to waves.
   B. Matter can be converted to particles.
   C. Matter exists as a particle and wave at the same time.
   D. Mass and energy can be seen as two names for the same underlying, conserved physical quantity.

4. 2 points Which of the following problem in physics wasn't fixed by quantum mechanics?
   A. The ultraviolet catastrophe of blackbody radiation
   B. The photoelectric mystery
   C. The helium spectrum mystery
   D. The contradiction between the universal speed of light and Galilean transforms.

5. 2 points Planck's quantization of energy was a desperate attempt to resolve the
   A. twin paradox.
   B. ultraviolet catastrophe.
   C. photoelectric mystery.

D. spectrum of hydrogen mystery.

6. 2 points We don't experience energy quantization in the everyday world because at our size

   A. the spaces between energy levels is too small to see with our senses.

   B. quantum mechanics isn't true on the macroscopic scale.

   C. energy becomes continuous past $n = 493, 523, 235, 674, 453, 213, 956$.

7. 2 points If I know the velocity of a subatomic particle precisely, then

   A. I know nothing about the particle's position.

   B. I known a very limited amount about the particle's position.

   C. The particle must be at rest.

   D. The particle can't be at rest.

Problems

8. 5 points An HCL molecule vibrates with a frequency of 8.1×10^{13} Hz. What is the smallest possible change of energy that this molecule could experience?

Solution:

$$\Delta E = hf = 6.63 \times 10^{-34}\text{Js}\left(8.1 \times 10^{13}\text{Hz}\right) = 5.4 \times 10^{-20}\text{J} = 0.34\text{eV}$$

9. 5 points How much energy is needed to ionize a hydrogen atom in the $n = 2$ state? What frequency photon would that require?

Solution:

$$E_{ion} = 0 - E_n = \frac{13.6eV}{n^2} = 3.4 \text{ eV}$$

10. [5 points] What percentage of mass does a hydrogen atom lose when it transitions from the $n = 3$ to the $n = 1$ state (in other words, $\Delta m/m_0$)? The rest mass of hydrogen is $m_0c^2 = 939 \times 10^6$ eV.

Solution:

$$\frac{\Delta m}{m_0} = \frac{\left(\frac{\Delta E}{c^2}\right)}{m_0} = \frac{\Delta E}{m_0c^2} = \frac{-13.6 \text{ eV}\left(1 - \frac{1}{3^2}\right)}{939 \times 10^6 \text{ eV}} = -1.29 \times 10^{-8}$$

11. Scientist A measures a quantum system by injecting an electron into a potential well, measuring its position with a laser beam (by finding where the photons reflect using Compton's equation, for example). After each measurement of its position, Scientist A flushes the electron out and repeats the measurement thousands upon thousands of times.

(a) [4 points] Each time the scientist locate the electron, will they see it as a wave or as a particle or something else?

Solution: They will see it as a particle in each individual study, but as a probablistic distribution they will see wave-like manifestations.

(b) [1 point] If the scientist were also to obtain some information about the momentum of the particle, what equation relates how accurately we can determine its momentum and position?

Solution:

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

(c) [5 points] After doing this thousands of times, the scientist finds that the probability of locating the particle in the well is:

$$P(x) = \frac{3}{8}x^2$$

What is the quantum wave function for the electron in this well, $\psi(x)$? *Just a note, this wave function isn't physically realistic.*

Solution:
Since $P(x) = \psi^2(x)$ for simple functions, then

$$\psi(x) = \sqrt{P(x)} = \sqrt{\frac{3}{8}}x$$

Again, it is important to note that this is a dummy function and isn't physicall reasonable.

(d) 5 points How long is the well? *Hint: Assume that the left side of the well starts at* $x = 0$ *and recall that the probability over the entire well has to equal 1.*

Solution:

$$\int_0^L \frac{3}{8}x^2 dx = \frac{3}{8}\frac{x^3}{3}\Big|_0^L = \frac{3}{8}\frac{L^3}{3} \quad \rightarrow L = 2$$

(e) 3 points Name three effects that we see in quantum mechanics that we don't see in our everday lives.

Solution:

12. 5 points An electron moves in a straight line with constant speed $v = 1.10 \times 10^6$ m/s which has been measured with precision of 0.10%. What is the best possible precision in which its position could be measured?

Solution: Recall that we require from the Heisenberg Uncertainty Principle that

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

Here we have,

$$\Delta p = m_e \Delta v = m_e(0.10)(1.10 \times 10^6 m/s) = m_e 1.10 \times 10^5 m/s$$

then, in the best case scenario,

$$\Delta x \Delta p = \frac{\hbar}{2}$$

$$\Delta x = \frac{\hbar}{2\Delta p}$$

$$\Delta x = \frac{\hbar}{2m_e 1.10 \times 10^5 m/s} = 5.26 \times 10^{-10} m$$

13. 5 points Suppose a particle is confined to the x-axis from $x = 0$ to $x = 3$, and that its wave function is given as $\psi(x) = Ax$. What is A? *Note, this isn't a realistic function, just one that is solvable without tables of integrals.*

Solution: We will use the **normalization** condition to solve for this:

$$\int_{-\infty}^{\infty} \psi(x)^2 = 1$$

$$\int_0^3 \psi^2(x)dx = \int_0^3 A^2x^2dx = \frac{A^2}{3}x^3 \mid_0^3 = 9A^2 = 1$$

Thus $A = 1/3$ and the full wave function is:

$$\psi(x) = \frac{x}{3}$$

14. 5 points Using the function from the previous problem, find the probability of finding the particle between $x = 1$ and $x = 2$.

Solution:

$$P(x \in (1,2)) = \int_1^2 \psi^2(x)dx = \int_1^2 \frac{x^2}{9}dx = \frac{x^3}{27} \mid_1^2 = \frac{8}{27} = 0.30$$